

Modeling Alpine Glaciers Melting using GEDI and ICESat-2 Altimetry data

Sofia Pelosi¹, Alireza Hamoudzadeh², Roberta Ravanelli³, Mattia Crespi^{2, 4}

1 Recent - *summa cum laude* M.Sc. graduate in Environmental Engineering, Sapienza University of Rome, Italy

2 Geodesy and Geomatics Division, DICEA, Sapienza University of Rome, Italy

3 Geomatics Unit, Department of Geography, University of Liège, Belgium

4 Sapienza School for Advanced Studies - Sapienza University of Rome, Rome, Italy

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Aim of the study – EO data

Glaciers are highly sensitive **indicators of climate change**

Continuous **monitoring** requires satellite remote sensing

Challenge: Field surveys are accurate, but costly and difficult to scale over large areas.

Proposed approach

Combine **freely available** satellite **LiDAR** observations within **one modelling framework**, without requiring dedicated field campaigns and **applicable anywhere** satellite coverage exists

Earth Observation data used

GEDI L2A

Full-waveform LiDAR

ICESat-2 ATL06

Photon-counting LiDAR

UAV DSMs 2020–2021

Independent
validation data

GEDI and ICESat-2 provide complementary LiDAR observations in terms of spatial sampling and acquisition strategy

The Rutor Glacier: a Representative Alpine Case Study

Located in the **north-western Italian Alps**, the **Rutor Glacier** is a **temperate Alpine glacier**, with distinct accumulation and ablation zones

8.4 km²

surface area

2,540–3,486 m

elevation range

3rd

largest glacier
in Aosta Valley

Research Objectives

01

Extract and pre-process
GEDI and ICESat-2
altimetry data for the
Rutor glacier

02

Develop **mathematical
model** of seasonal
glacier surface height
variation

03

Estimate model
parameters via **non-linear**
Levenberg damped Gauss-
Newton **least squares**

04

Validate results with
photogrammetric DSM
interpret ablation trends

Why Rutor?



Dual satellite coverage

Strong **overlap** of **GEDI** and **ICESat-2** tracks



Representative dynamics

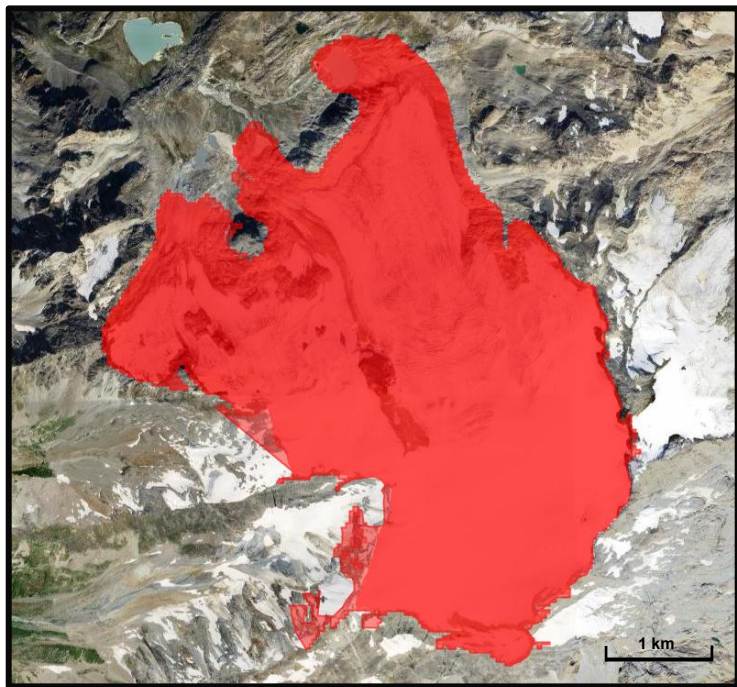
Clear **ablation/accumulation** zones



Independent validation

High-resolution UAV DSMs available for **2020** and **2021**

Data extraction and harmonization



Rutor ROI, from the GLIMS glacier inventory

GEDI Extraction & Quality Filtering (GEE)

GEDI L2A elevation product within the GLIMS ROI; **quality control** procedures application; EGM2008 geoid correction to **orthometric heights**

ICESat-2 Acquisition & Geodetic Harmonization

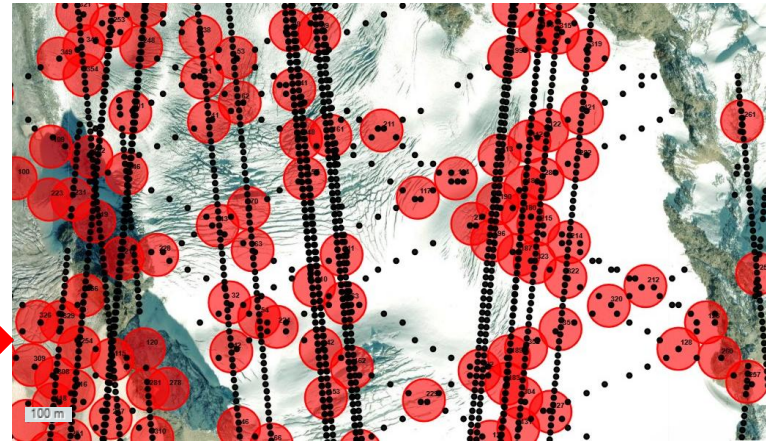
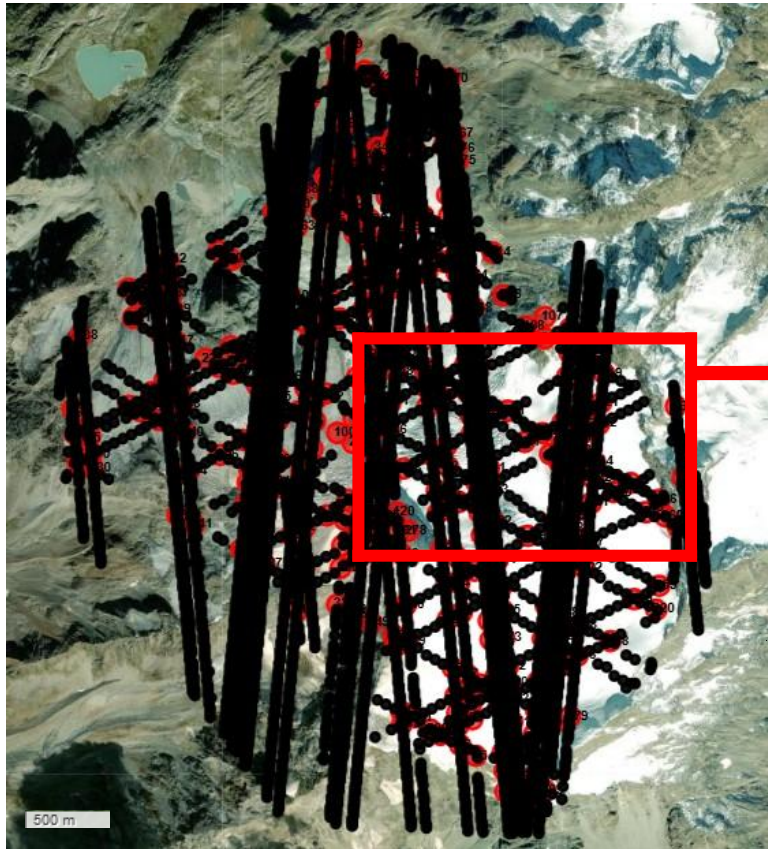
ATL06 data retrieved via earthaccess within a **convex-hull mask** around the GEDI footprints; ellipsoidal heights converted to the **same orthometric datum as GEDI**

DEM & DSM Integration

Copernicus GLO-30 and ALOS AW3D30 for **slope, aspect and hillshade**; two high-resolution **UAV DSMs** (2020, 2021) as **independent validation data**

Output: daily GEDI and ICESat-2 CSV surface elevation files, spatially matched and referenced to a common vertical datum — ready for the vectorial reconstruction described next

From orbital tracks to Anchor Points



Anchor Points convert sparse, non-repeating satellite footprints into a fixed network of locations suitable for continuous time-series reconstruction

Track reconstruction

Sequential **footprints of same dates** connected into LineStrings



Intersection detection

Crossings between tracks from different dates identified as **nodes**



DBSCAN clustering

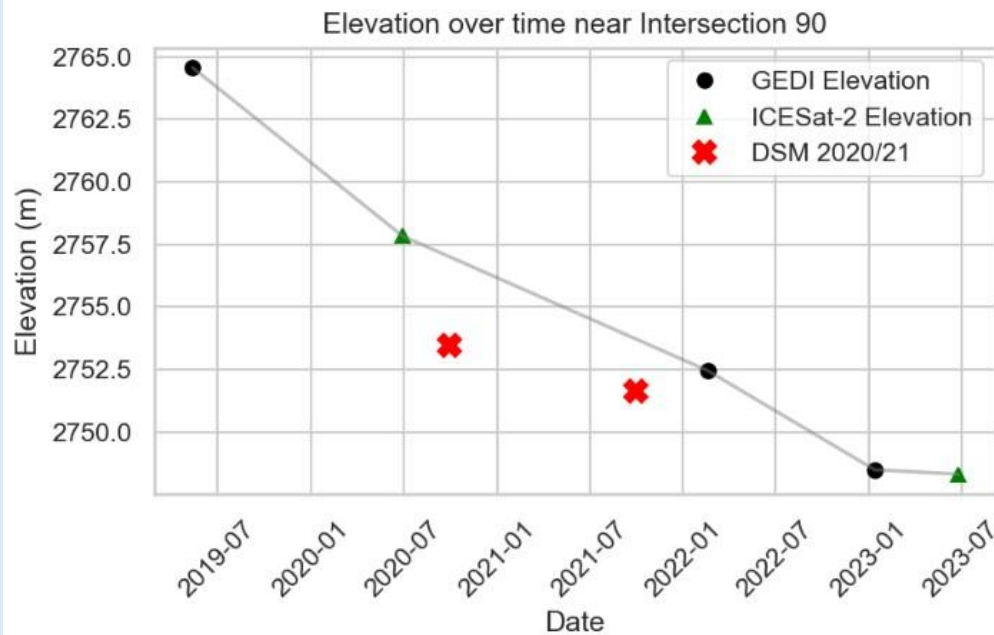
Nearby **intersections grouped** ($\epsilon \approx 50-60$ m)



Anchor Points

Cluster centroids retained as stable locations

Building Continuous Elevation Time Series



Example: reconstructed elevation time series at Anchor Point 90, combining GEDI, ICESat-2 and DSM 2020/21 observations.

Spatial Buffering

A ~60 m **circular buffer** is defined around each Anchor Point **centroid**

Radial Averaging per Date

Footprints within the buffer on a given acquisition date are **averaged** into a single elevation value

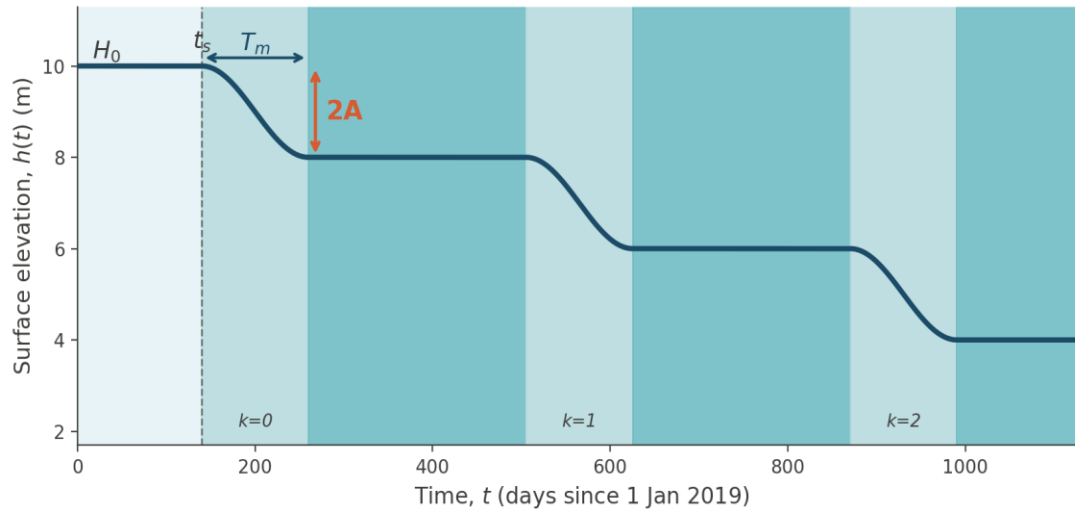
DSM Validation Points

The 2020 and 2021 **UAV DSMs** contribute two additional, **precise elevation** values, fixed at the **end** of each **hydrological year** (30 September)

*Continuous elevation time series construction at stable geographic points,
the direct input for the seasonal ablation model*

A piecewise seasonal melt model – parameters and variables

Piecewise seasonal elevation model



- Elevation follows a **repeating seasonal pattern**: cosine-shaped melt drop, then flat winter stabilization
- Each melt season steps down cumulatively from where the previous winter ended ("**step-down**" effect)
- Simplified model with **no spatial dependency**

Parameters and variables

Global parameters

A (m): melt half-amplitude
 t_s (d): melt onset
 T_m (d): melt duration

Local parameter

$H_{0,i}$ (m):
 baseline elevation for each
 Anchor Point

Auxiliary variables

t (d): continuous time
 k annual melt-cycle index
 $Y = 365.25$ days

Shared vs. local

A , t_s and T_m are common to all locations; $H_{0,i}$ varies by Anchor Point.

Piecewise function: three phases

Initial phase

$$t < t_s$$

$$h(t) = H_0$$

Active ablation

$$t_s + kY \leq t < t_s + kY + T_m$$

$$h(t) = H_0 - 2kA + A \left[\cos\left(\frac{\pi}{T_m}(t - (t_s + kY))\right) - 1 \right]$$

Winter stabilization

$$t_s + kY + T_m \leq t < t_s + (k+1)Y$$

$$h(t) = H_0 - 2(k+1)A$$

Physical constraints

$$A \geq 0$$

Melt amplitude **cannot be negative**, it must always represent a **reduction in elevation**, never accumulation

Unilateral

$$0 \leq T_m \leq 365$$

Melt duration **cannot be negative** and **cannot exceed** a full calendar year

Bilateral

$$0 \leq t_s \leq 200$$

Melt onset **cannot precede 1 Jan 2019** or **exceed ~200 days** (mid-July)

Bilateral

- Constraints apply **only to the three global parameters** (A , t_s , T_m) — local baselines $H_{0,i}$ are left unconstrained, since any real elevation is physically admissible
- Any of the 9 **optimization** runs whose converged parameters fall **outside these bounds** is **discarded** before RMSE-based best-fit selection

From a Nonlinear Fit to a Validated Result

The problem is solved as a nonlinear least-squares fit, iteratively linearized and solved. Once converged, the same framework lets us quantify how much to trust the result — and check it against independent data

Parameter uncertainty

- **A-posteriori variance factor** s used to provide **statistical uncertainty** for parameters
- The **main diagonal** of the **covariance matrix** $C_{\hat{x}}$, obtained by **scaling** the **cofactor matrix** $Q_{\hat{x}}$ **with** s , contains the **variances** of the estimated **parameters**; the **square root** yields the **standard deviation**
- **Large** $\sigma_{\hat{x}_k}$ flags a parameter that is only **weakly constrained** by the data

External DSM validation

- **Model evaluated** at the **DSM acquisition dates** for a total number of validation points equal to N_{DSM}
- Computation of **vertical error** between the model prediction and DSM
- Summarized as **Mean Error**, **standard deviation** σ_{ε} , and **$RSME_{DSM}$**

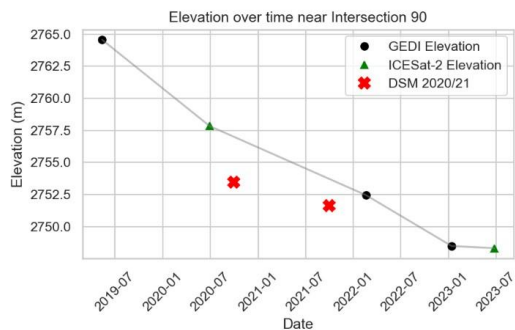
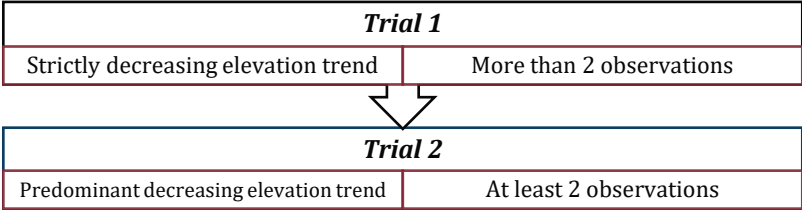
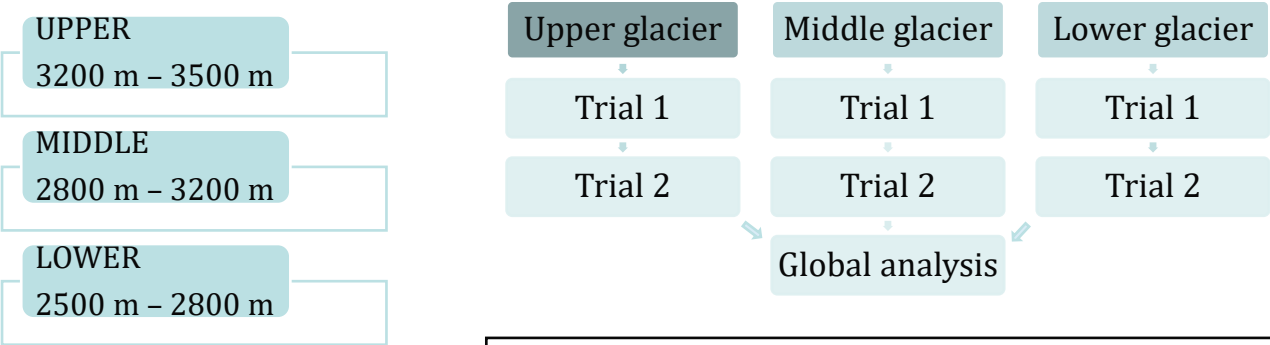
Why joint estimation works

*Estimating a single set of global parameters across many pooled intersections keeps the **system strongly overdetermined** ($m \gg 3 + N$), **resolving the sparse-data problem** that would affect any individual **Anchor Point** analyzed **in isolation**.*

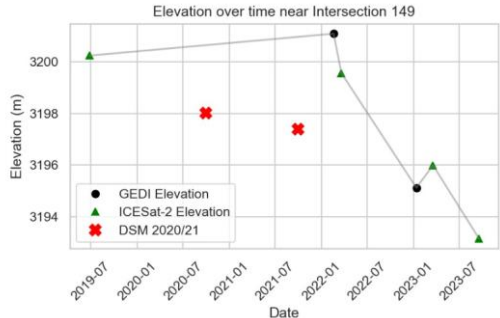
Joint Analysis Setup: Sectors, Trials and Initial Configurations

Glacier sector subdivision

based on available observations



Elevation timeseries with strictly decreasing trend – Example intersection ID 90



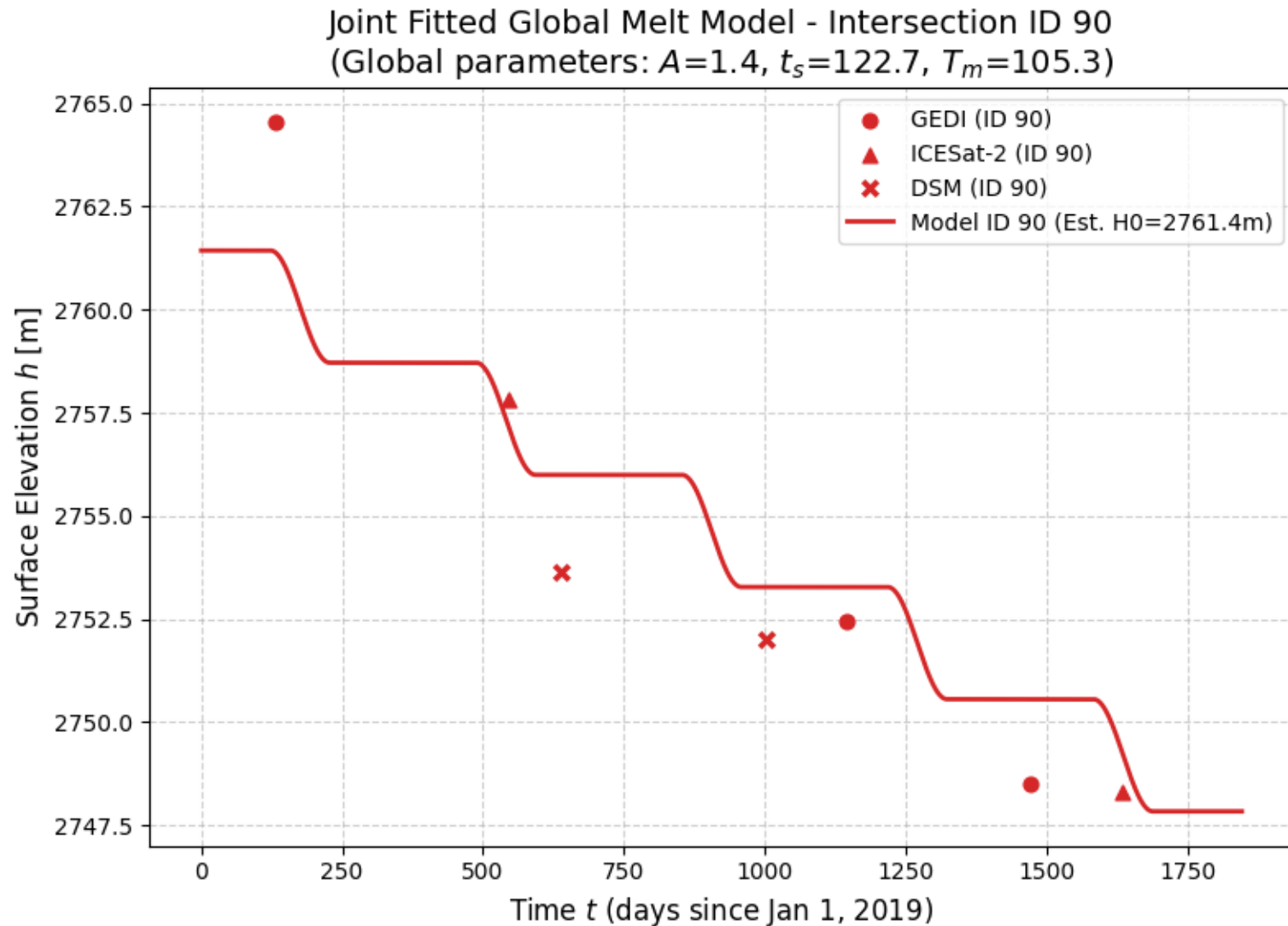
Elevation timeseries with predominant decreasing trend – Example intersection ID 149

9 initial configurations

$A_0(m)$		
1.5	2.0	2.5
	$t_{s,0}(d)$	$T_{m,0}(d)$
A	135	165
B	150	150
C	165	135

Optimization with **lowest RMSE** while satisfying **physical constraints** is retained for following results

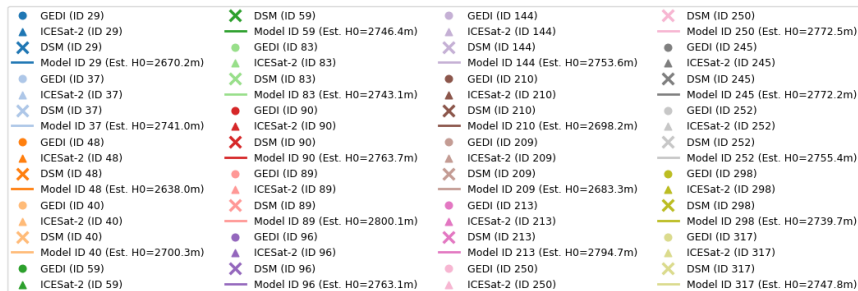
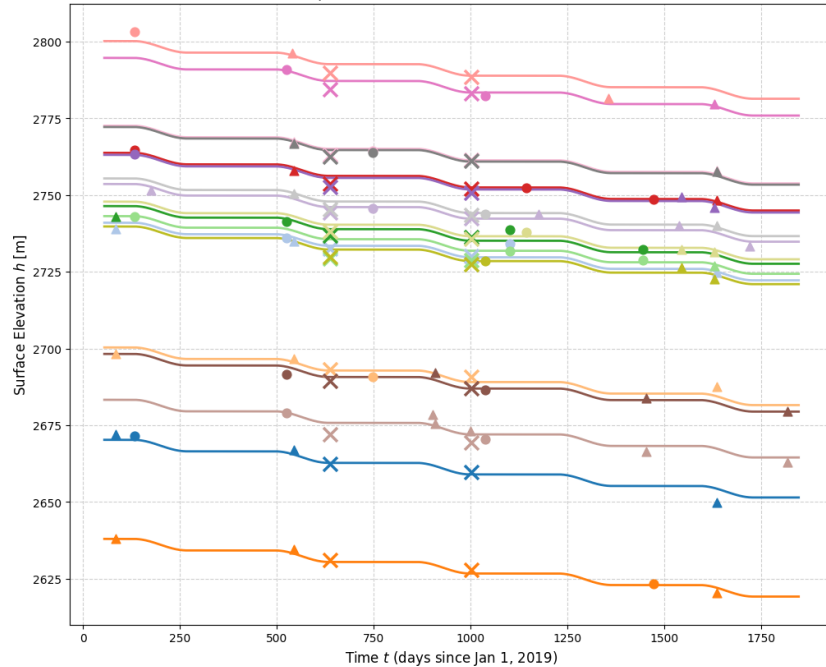
Example: Fitted Curve for a Single Anchor Point (ID 90)



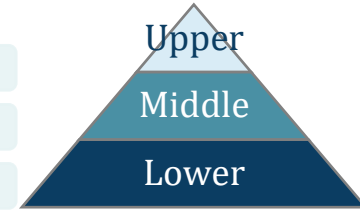
First and second joint analysis results for Upper - Middle - Lower sectors

Example First - Lower

First Joint Fitted Melt Model (Lower Part) - Best Fit Only
(Global parameters: $A=1.9$, $t_s=129.9$, $T_m=139.0$)



$A \uparrow$	Toward Lower sector
$t_s \downarrow$	Toward Lower sector
$T_m \uparrow$	Toward Lower sector



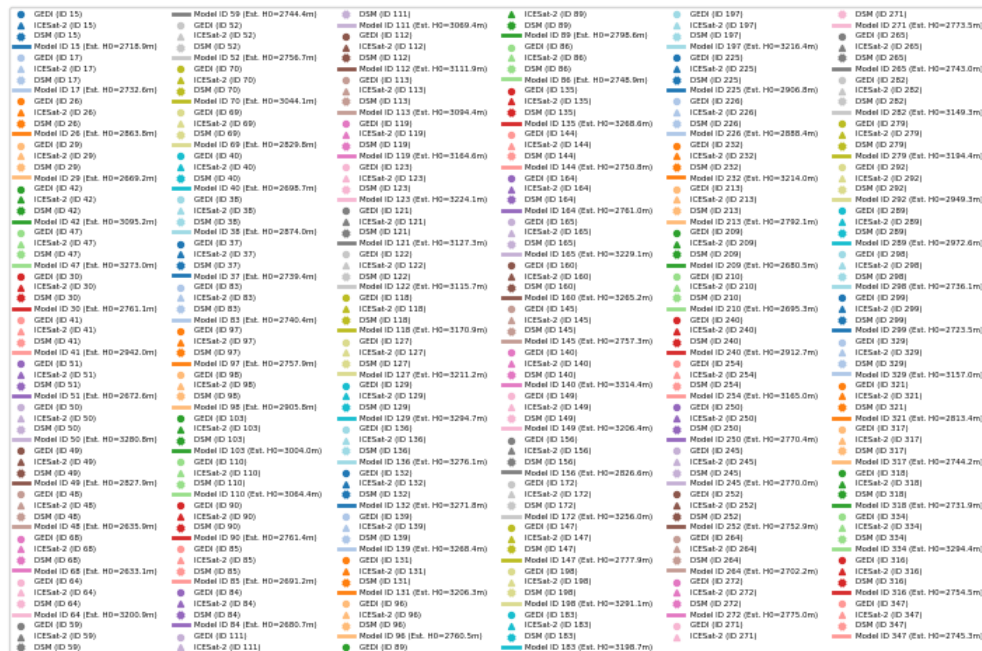
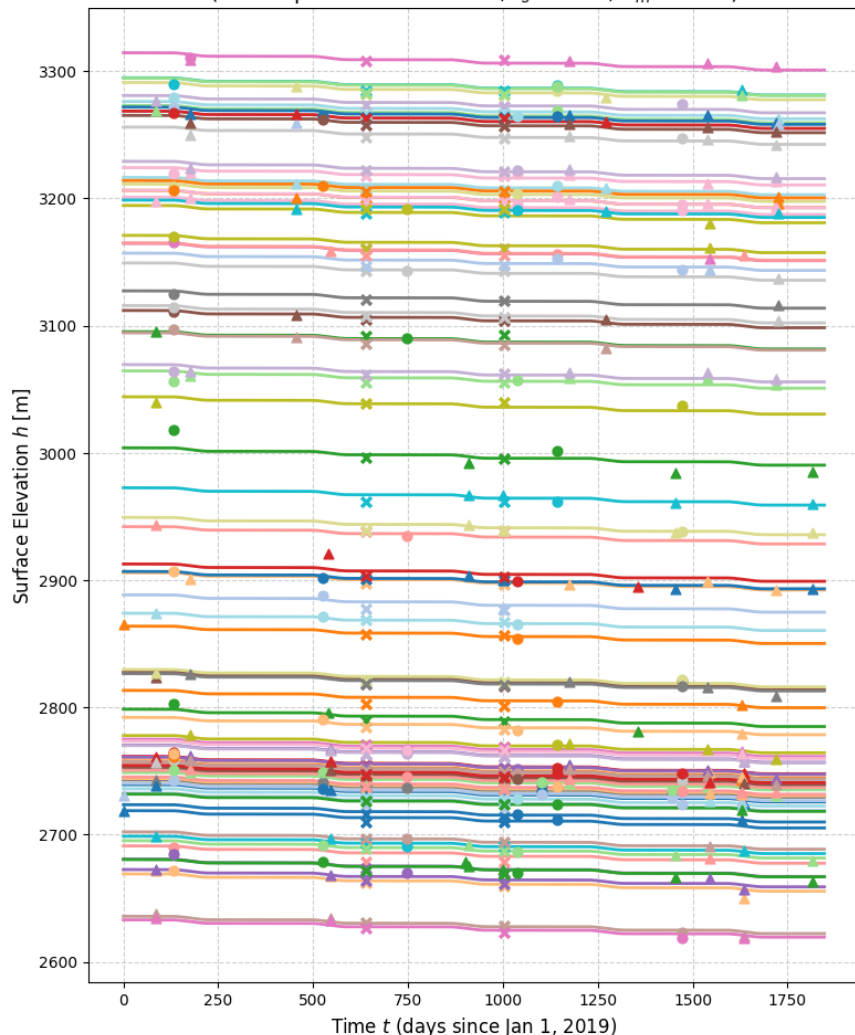
FIRST	Upper	Middle	Lower
A (m)	1.0	1.4	1.9
t_s (d)	118.5	162.2	129.9
T_m (d)	137.2	123.1	139.0

SECOND	Upper	Middle	Lower
A (m)	0.9	1.2	1.8
t_s (d)	104.9	144.3	140.8
T_m (d)	104.0	120.4	119.4

 Consistent with expected trend	 Inconsistent with expected trend
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Global joint analysis results

Joint Fitted Global Melt Model - Best Fit Only
(Global parameters: $A=1.4$, $t_s=122.7$, $T_m=105.3$)



	Second Upper	Global	Second Lower
A (m)	0.9	1.4	1.8
t_s (d)	104.9	122.7	140.8
T_m (d)	104.0	105.3	119.4

Each parameter estimate is a compromise between Lower and Upper estimates from the Second joint analysis

Goodness-of-fit: sector comparison

Analysis	RMSE (m)	Max res. (m)	n = parameters	m = obs.	v = d.o.f.	T = test statistic	K = critical threshold	s = a-posteriori variance factor (m ²)	σ = real data noise = $\sqrt{s}$ (m)
Upper – 1st	2.2	- 5.5	13	33	20	159.5	31.4	8.0	2.8
Middle – 1st	3.5	11.5	11	25	14	312.1	23.7	22.3	4.7
Lower – 1st	1.6	- 4.6	21	71	50	192.0	67.5	3.8	1.9
Upper – 2nd	2.0	5.8	22	70	48	274.6	65.2	5.7	2.4
Middle – 2nd	3.5	14.9	33	94	61	1132.8	80.2	18.6	4.3
Lower – 2nd	1.7	- 5.1	42	137	95	409.8	118.7	4.3	2.1
Global	2.8	13.8	91	301	210	2367.1	244.9	11.3	3.4

Key points

- **Upper** and **Lower** sectors consistently **outperform Middle** (RMSE < 2.2 m vs 3.5 m)
- **Global** analysis performs **between** the best and worst **sector-specific** results
- **OMT** is **always rejected** before rescaling — the magnitude of **s**, not the binary test outcome, indicates **precision**
- **Middle** sector: **lowest** data **quality/quantity**, especially in the **first trial**

Parameter uncertainty analysis – melt-half Amplitude

Analysis	σ_A (m)
Upper – 1st	0.2
Middle – 1st	0.4
Lower – 1st	0.1
Upper – 2nd	0.1
Middle – 2nd	0.2
Lower – 2nd	0.1
Global	0.1

Key points

- The **parameter uncertainty** (σ_A) is typically 8–20% of the estimated A , confirming the **amplitude is well constrained** by the observations
- **Middle – 1st** is the clear **exception** ($\sigma_A = 0.4$ m), consistent with its poor RMSE and high variance factor s (Section 5.3)

σ_{t_s} and σ_{T_m} are omitted: winter-biased sampling leaves too few melt-season observations to constrain them individually

DSM statistical validation

Results for vertical errors between the model prediction and DSM

Analysis	NDSM = tot validation points	ME (m) = Mean Error	σ_{ε} (m) = Standard Deviation	RMSE _{DSM} (m) = Root Mean Square Error
Upper – 1st	20	-1.3	1.6	2.1
Middle – 1st	16	-1.8	1.9	2.6
Lower – 1st	36	-1.2	1.6	2.0
Upper – 2nd	38	-1.4	1.6	2.1
Middle – 2nd	60	-1.1	2.5	2.8
Lower – 2nd	78	-1.2	1.7	2.1
Global	176	-1.3	2.1	2.5

Key points

- **Mean error is small** and consistently **negative** (–1.1 to –1.8 m): the **model** slightly **overestimates** surface elevation across all sectors
- The **Middle sector**, worst in-sample, **matches** the other sectors once **validated against the DSM** — its poor in-sample fit reflects **noisy input data**, not a badly reconstructed curve
- Validation **RMSE** converges to a narrow **2.0–2.8 m** range across all seven analyses, **regardless of sector or dataset size**

Conclusions: main findings & limitations

Main findings

Increasing dataset improves accuracy. **Spatial subdivision** too but not uniformly: Upper (2.0 m), Lower (1.7 m) and Middle (3.5 m) RMSE. **Global RMSE** (2.8 m) is a **compromise** of second sector specific analysis

Estimated **parameters** are **glaciologically consistent**: A and T_m increase from Upper to Lower as expected, t_s remains a persistent **exception**

Reconstructed curves **agree** with **independent UAV-derived DSMs**, supporting the reliability of the satellite-based reconstruction

Limitations

Active **melting phase sampling scarcity**: winter-heavy observations constrain A well, but leave T_m and t_s poorly determined

No accumulation logic: the model describes ablation only, restricting analysis to consistently downward-trending intersections

Uniform GEDI/ICESat-2 weighting: Q_{yy} assumed as identity, ignoring the sensors' different precision, a likely driver of the elevated variance factors s

These are methodological, not conceptual, limitations — they define the boundary of the simplified model and directly motivate the future developments

Future developments

1 Complete model & terrain-dependent parameters

Let A , t_s and T_m be **terrain-dependent** and **vary** explicitly **with elevation** (and slope, aspect, hillshade)

2 Comprehensive regional surface scaling

Extend estimation beyond track-intersection Anchor Points to exploit the full footprint density over the **entire glacier surface**

3 Sensor-specific stochastic weighting

Replace $Q_{yy} = I$ with a diagonal matrix of **sensor-specific a priori** variances for GEDI and ICESat-2

4 Automated error handling & outlier detection

Integrate a dedicated statistical pipeline for anomaly and **outlier detection** ahead of the optimization

THANK YOU FOR THE ATTENTION!