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DEPARTMENT OF MATHEMATICS
DEPARTMENT OF EXCELLENCE 2023-2027

MODELLISTICA E CALCOLO SCIENTIFICO
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ASI Agenzia Spaziale Italiana

Remote Monitoring of Ground Displacement via Functional Time **Series and Conformal Inference**

T. Bortolotti, F. Casu, M. Virelli, D. Tapete, M. Siciliani de Cumis, A. Menafoglio, S. Vantini

Workshop ASI
"Tecnologie satellitari e analisi multi-rischio: l'esperienza dei progetti I4DP_SCIENCE e prospettive future"

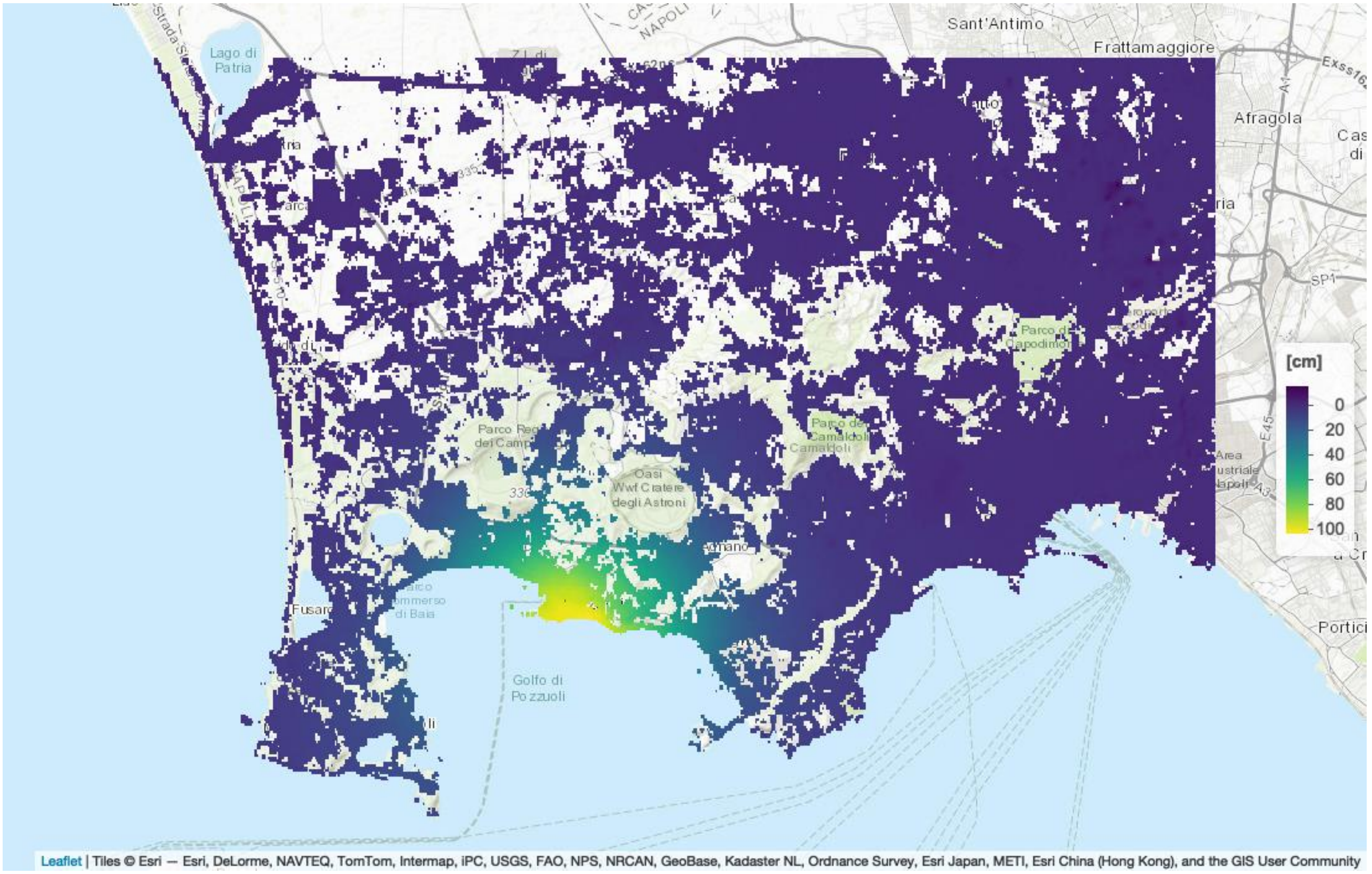
27 May 2025

High-dimensional interferometric data

SAR data are high-resolution images collected by the satellites orbiting around the Earth.

Advanced Interferometric procedures convert raw SAR data into **very rich and accurate** information on how the ground moves over time.

Differential SAR interferometry (DInSAR) has key potential for natural hazards prevention over large regions.



Vertical ground displacement over the Phlegraean Fields, Italy, on May 11, 2024, in coherent pixels, cumulated with respect to the reference time March 16, 2015.

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Temporal series of vertical ground displacement over the Phlegraean Fields, Italy, from November 16, 2020, until May 11, 2024. The vertical ground displacement is cumulated with respect to the reference time of March 16, 2015, and is only displayed in coherent pixels.

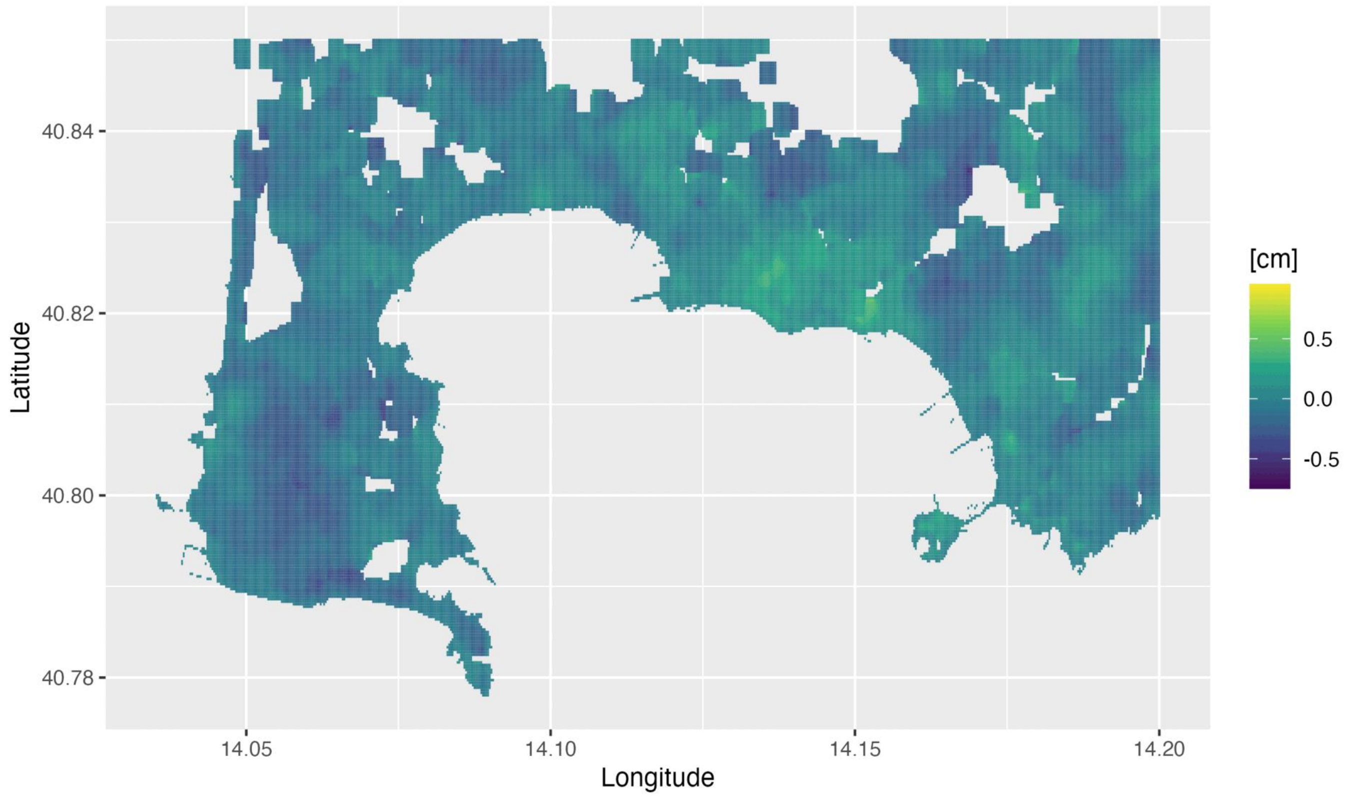
Nonparametric statistical monitoring of ground displacement

General methodological framework

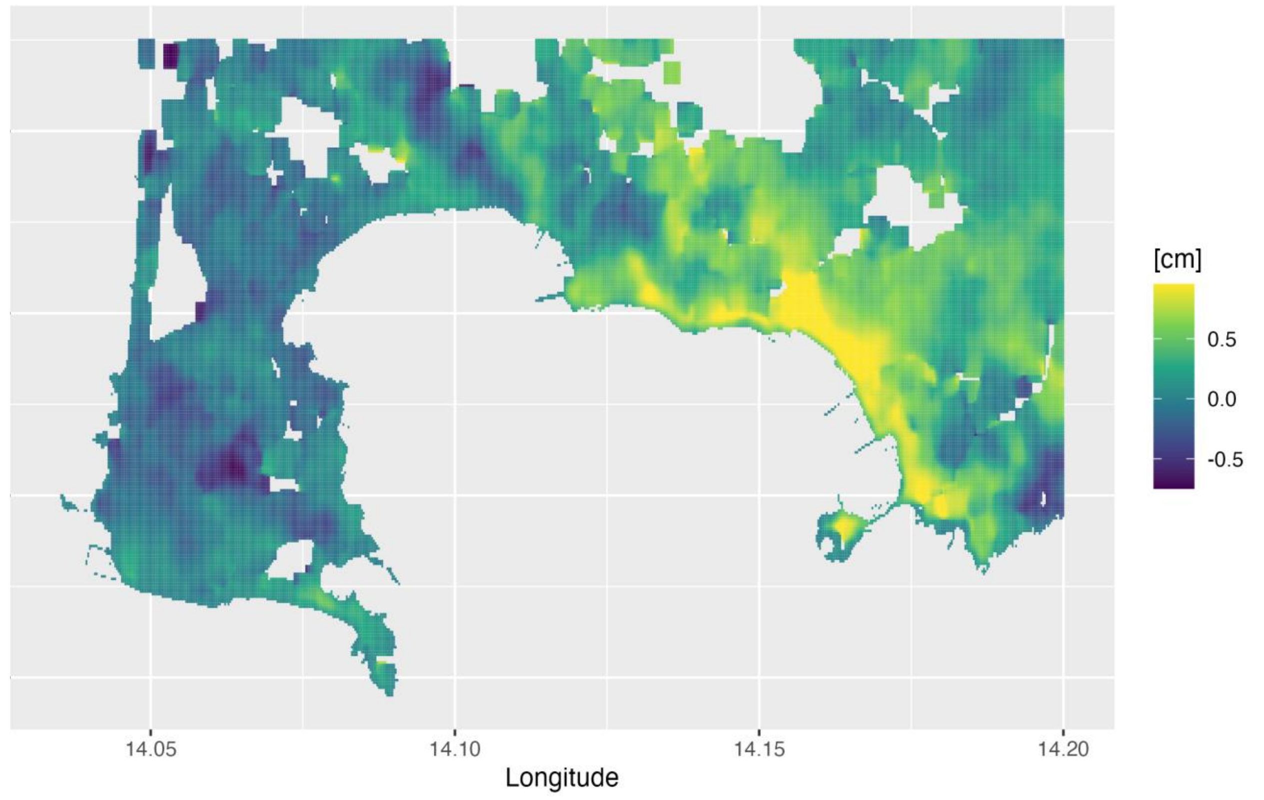
$Y_1, \ldots, Y_n$ time series s.t. $Y_i : \Omega \to L^2(D), D \in \mathbb{R}^2$.

Y_{n+1} is a new observation of the time series.

Historical information



New observation



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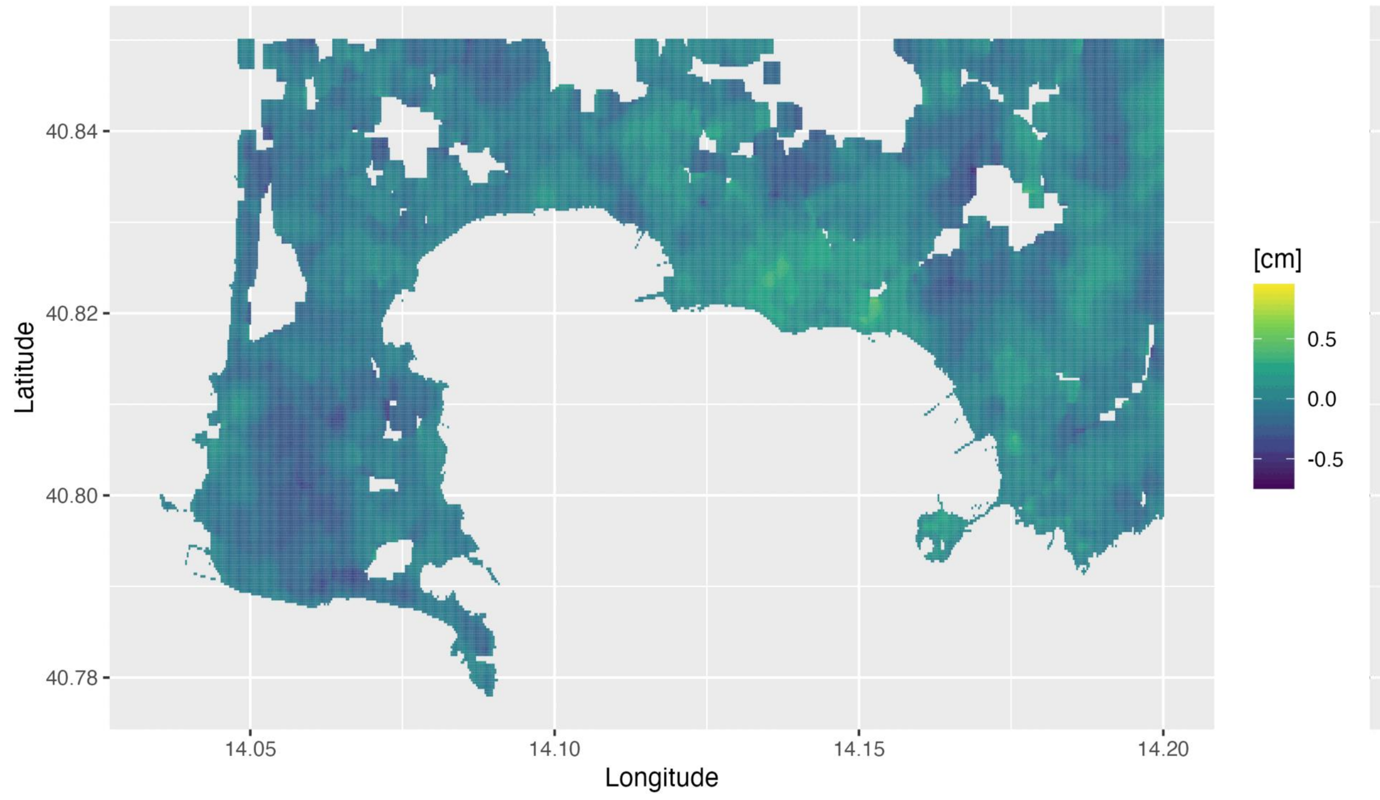
Conformal prediction band at level α

$$C_{1-\alpha}(Y_{1:n}) = \{y \in L^2(D): y(u,v) \in [\hat{\mu}(u,v) \pm Q_{1-\alpha}(u,v)], \forall (u,v)\}$$

- $\hat{\mu}(u,v)$ point predictor at coordinates (u,v),
- $Q_{1-\alpha}(u,v)$ threshold at (u,v) calibrated for α.

Historical information



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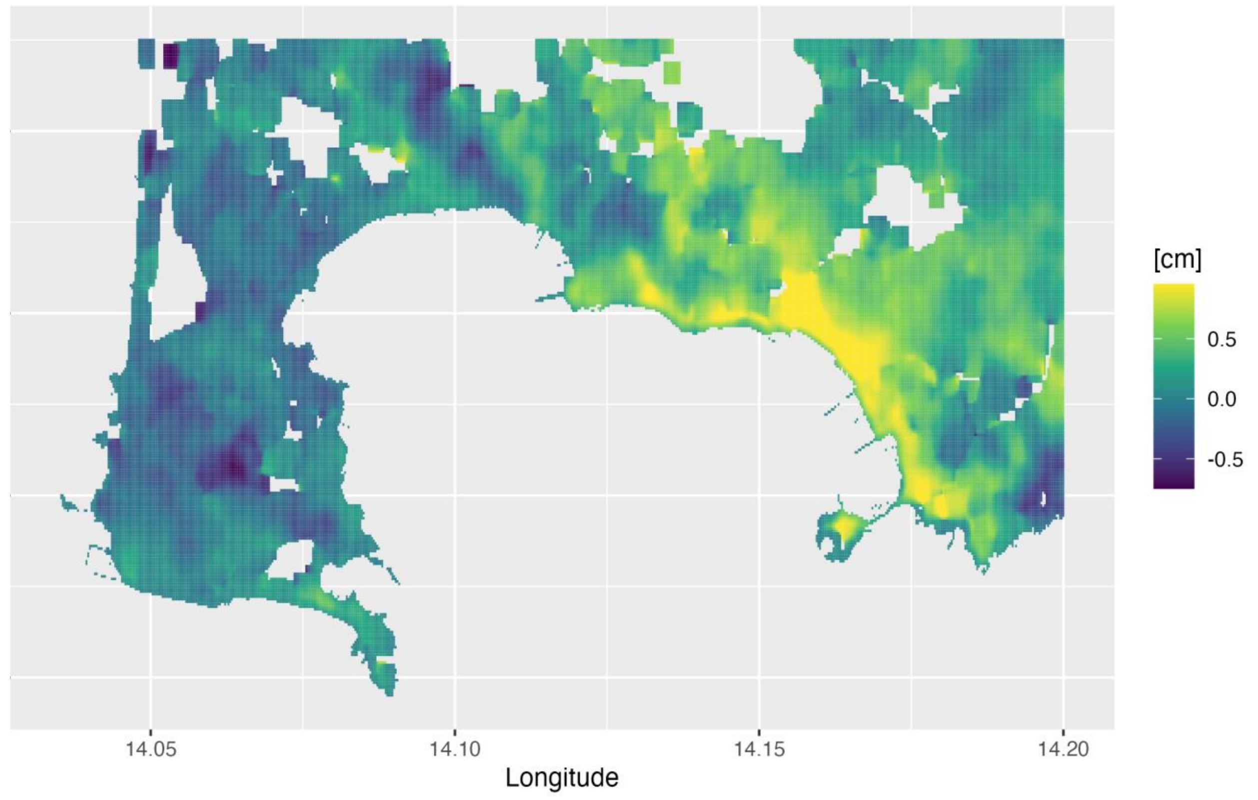
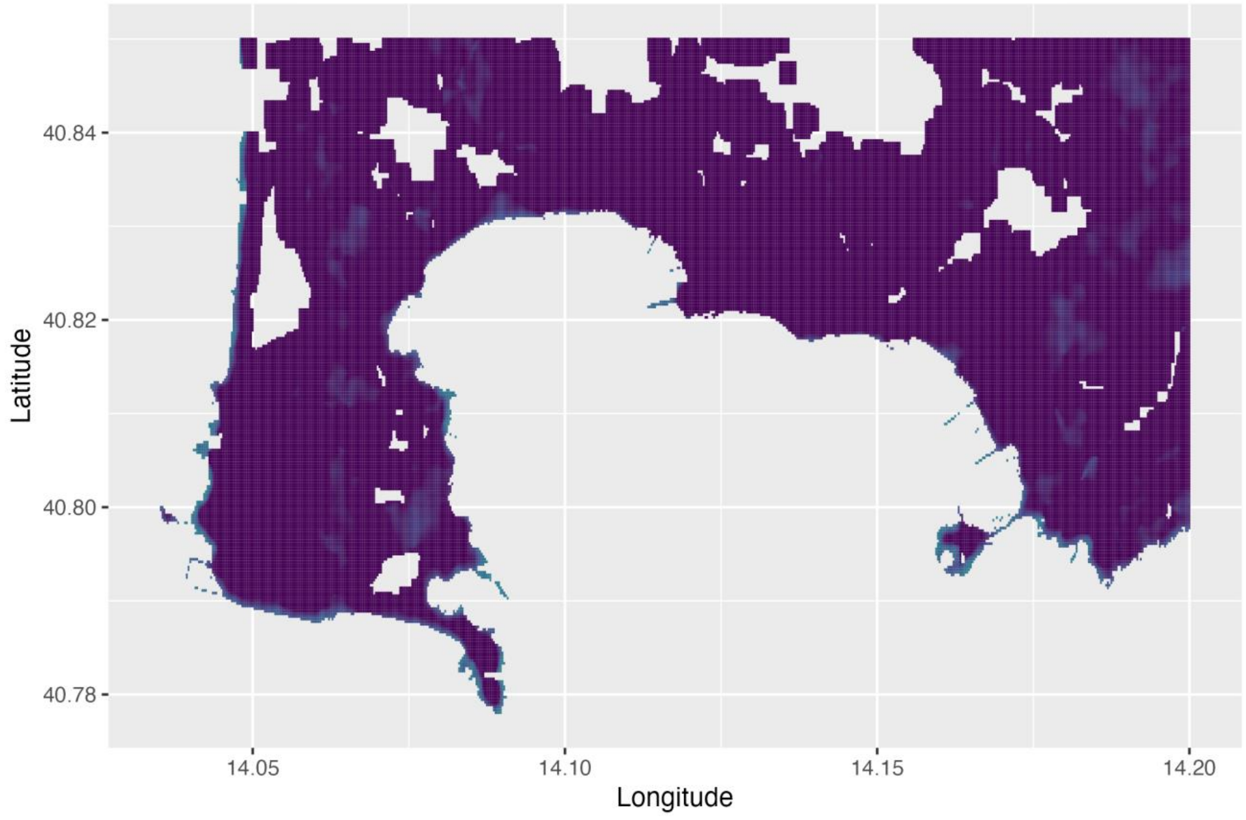
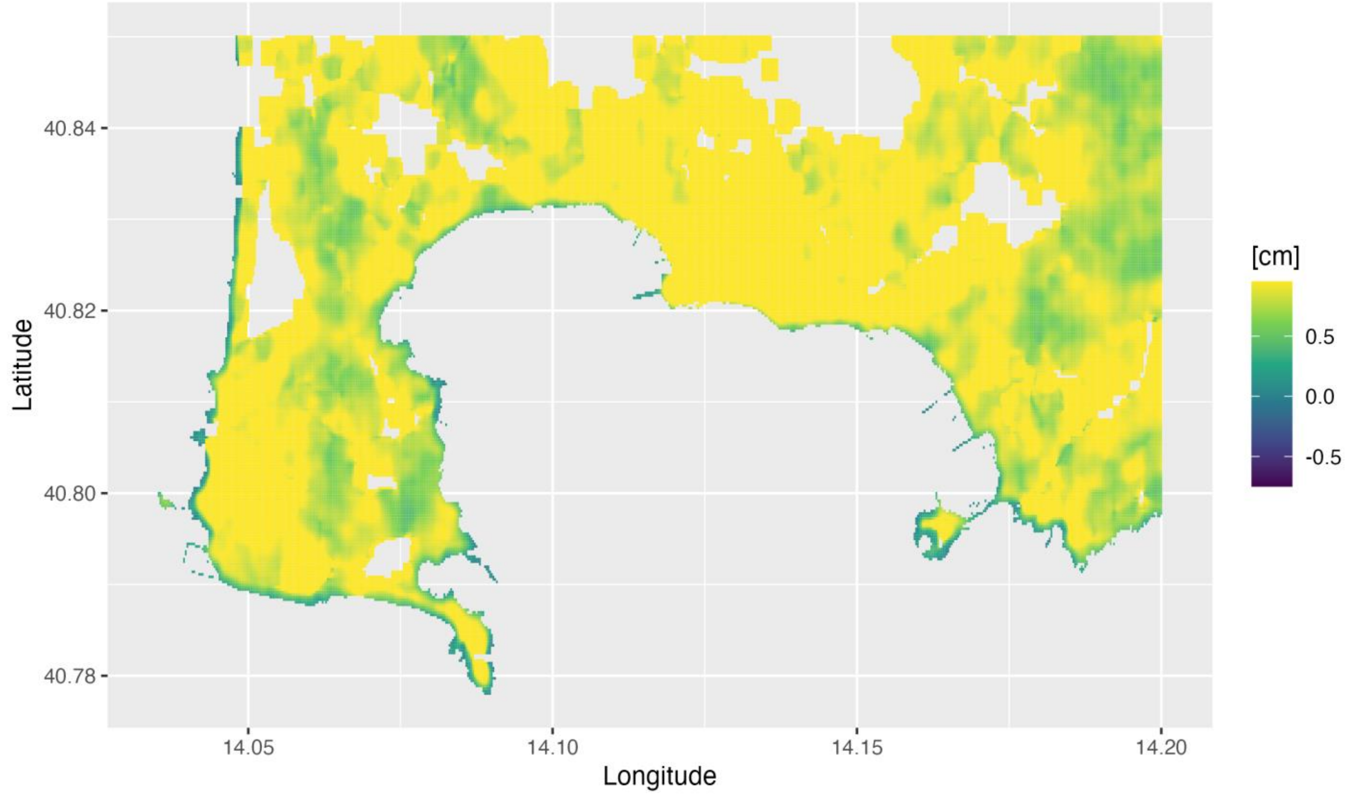


Lower bound at 10%



Upper bound at 10%



A dicotomic warning map

Conformal dicotomic warning map

Given a history $Y_1, \ldots, Y_n$ and a new observation Y_{n+1}, the *conformal dicotomic warning map* for Y_{n+1} is

$$w_{n+1}(u, v) = \begin{cases} 0, & Y_{n+1} \in C_{1-\alpha}(Y_{1:n}, ; u, v) \\ 1, & Y_{n+1} \notin C_{1-\alpha}(Y_{1:n}, ; u, v) \end{cases}$$

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Guarantees

Thanks to the conformal guarantees, we have that when $\mathrm{Y}_{\mathrm{n}+1}$ is not an anomaly

$$\mathbb{P}[\exists\ (u,v) : \mathrm{w}_{\mathrm{n}+1}(\mathrm{u},\mathrm{v}) = 1] \leq \alpha$$

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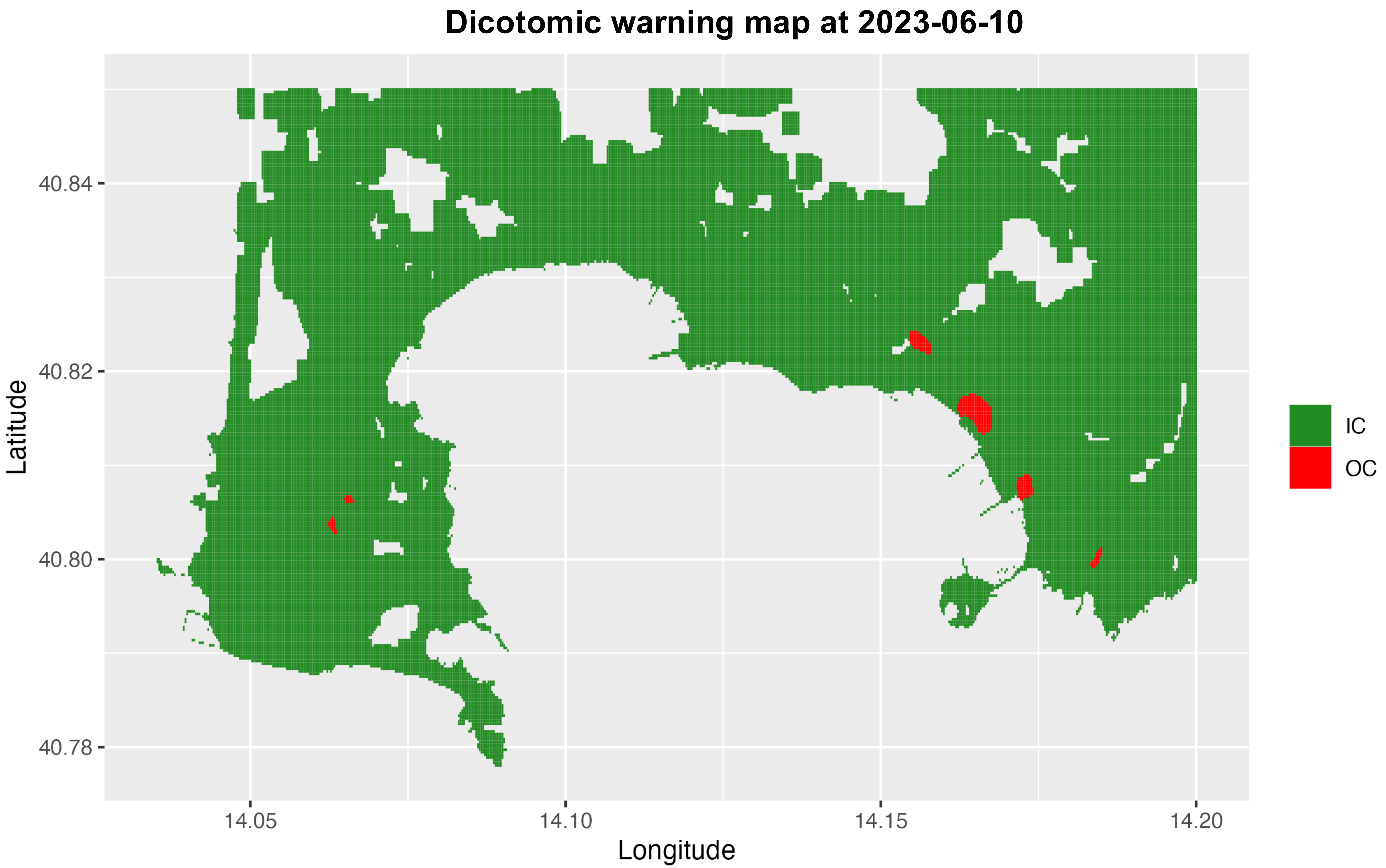
Main advantages

- No assumption on the data distribution
- Localization of the anomalies
- Control the risk of false warnings

Dicotomic warning map



Dicotomic warning map for June 10, 2023. The pixels where the map takes value 1 are considered as anomalies and are colored in red. The remaining pixels are considered as in-control.
From Bortolotti, Casu, Virelli, Tapete, Siciliani de Cumis, Menafoglio, Vantini (2024). Congress Proceedings of the 75th International Astronautical Congress.

P-value warning map

Fix the point (u, v) and consider a univariate setting.

Inspiration: Classical point-wise p-value
Given a history $\mathrm{Y}_1(u, v), \dots, \mathrm{Y}_\mathrm{n}(u, v)$, the point-wise p-value of a new observation $\mathrm{Y}_{\mathrm{n}+1}(u, v)$ is

$$
\begin{aligned}
&p_{n+1}(u, v) = \max_{\alpha} \alpha \\
&\mathrm{s.t.} \quad Y_{n+1}(u, v) \in PI_{1-\alpha}(Y_{1:n}(u, v))
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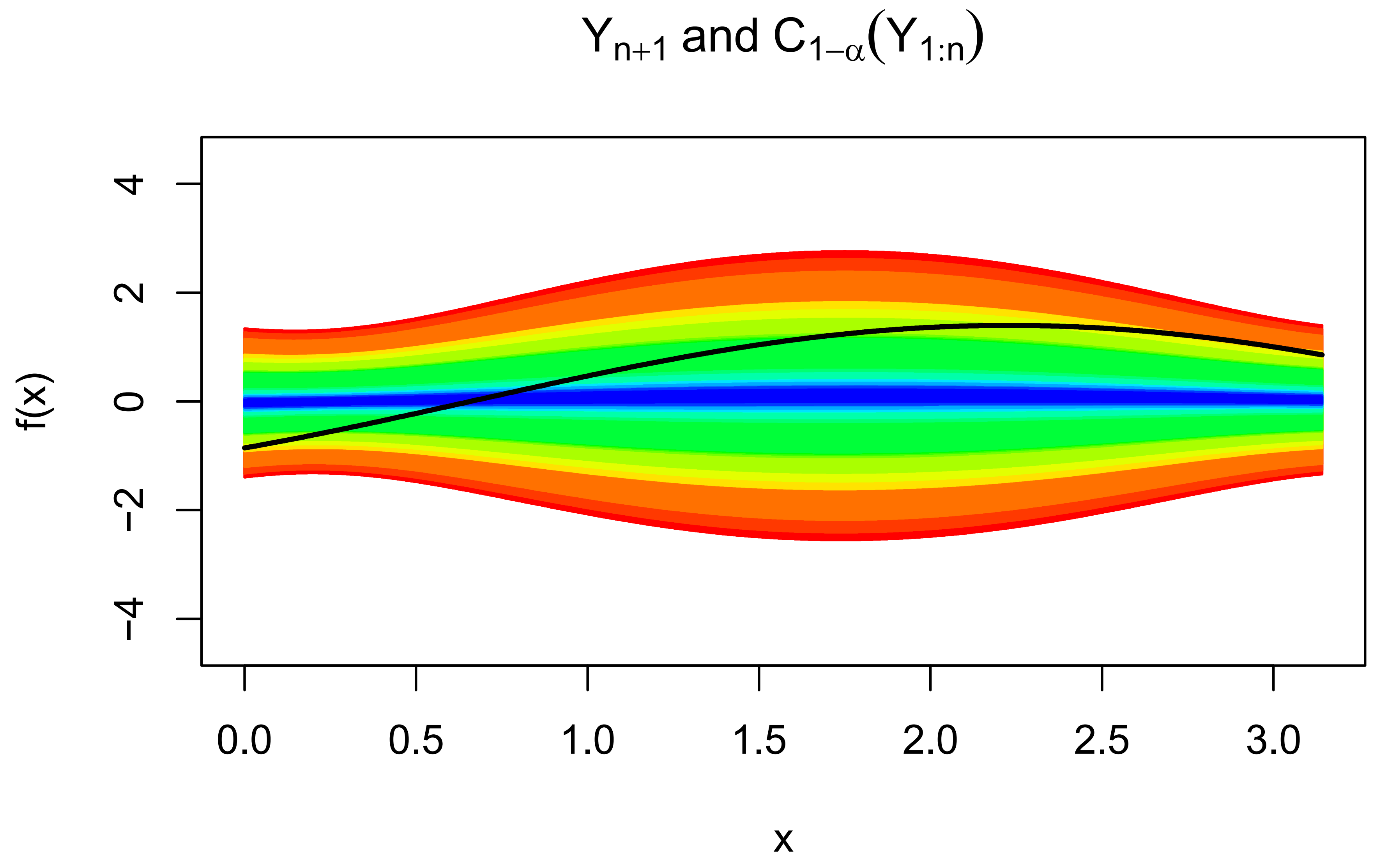

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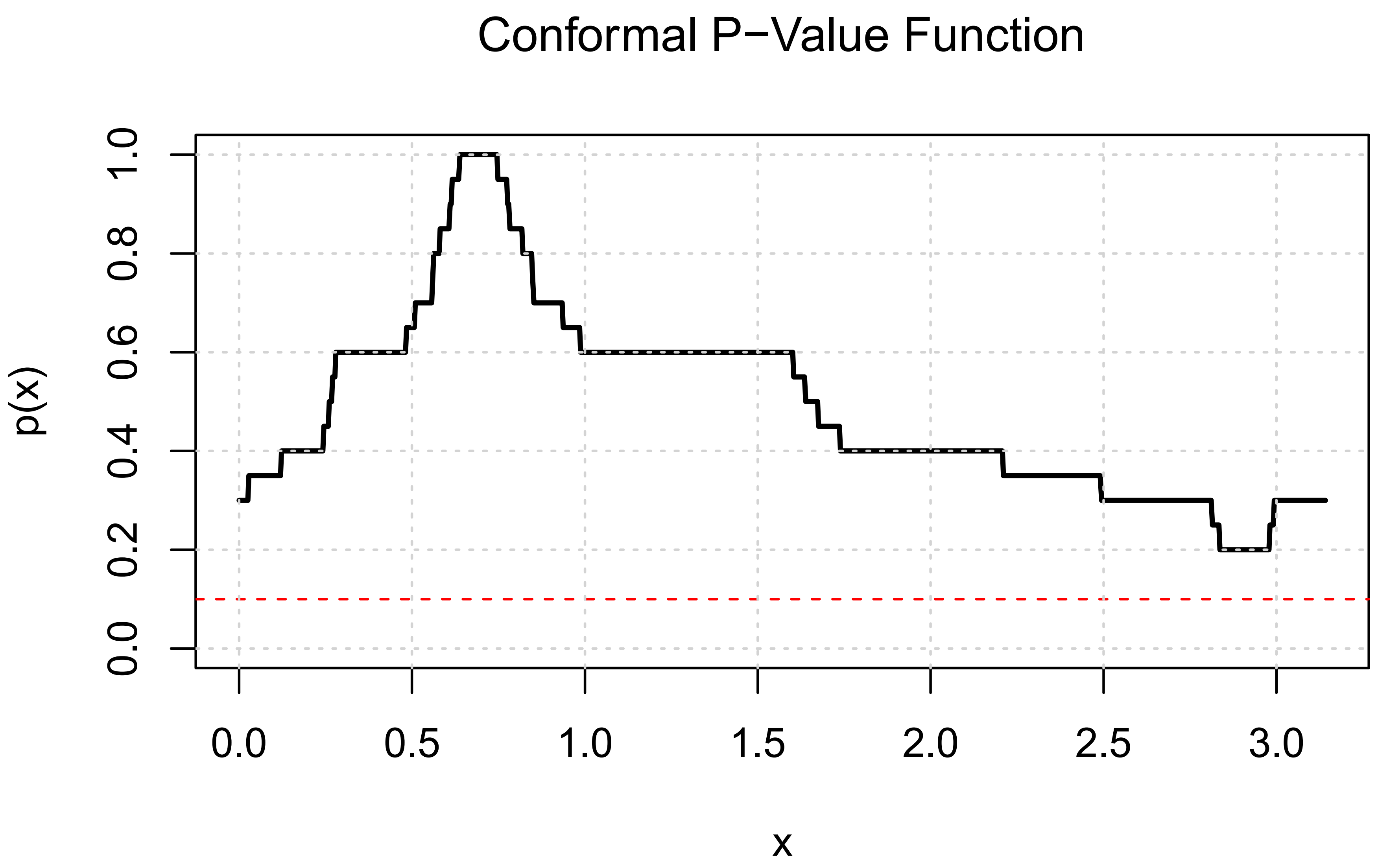

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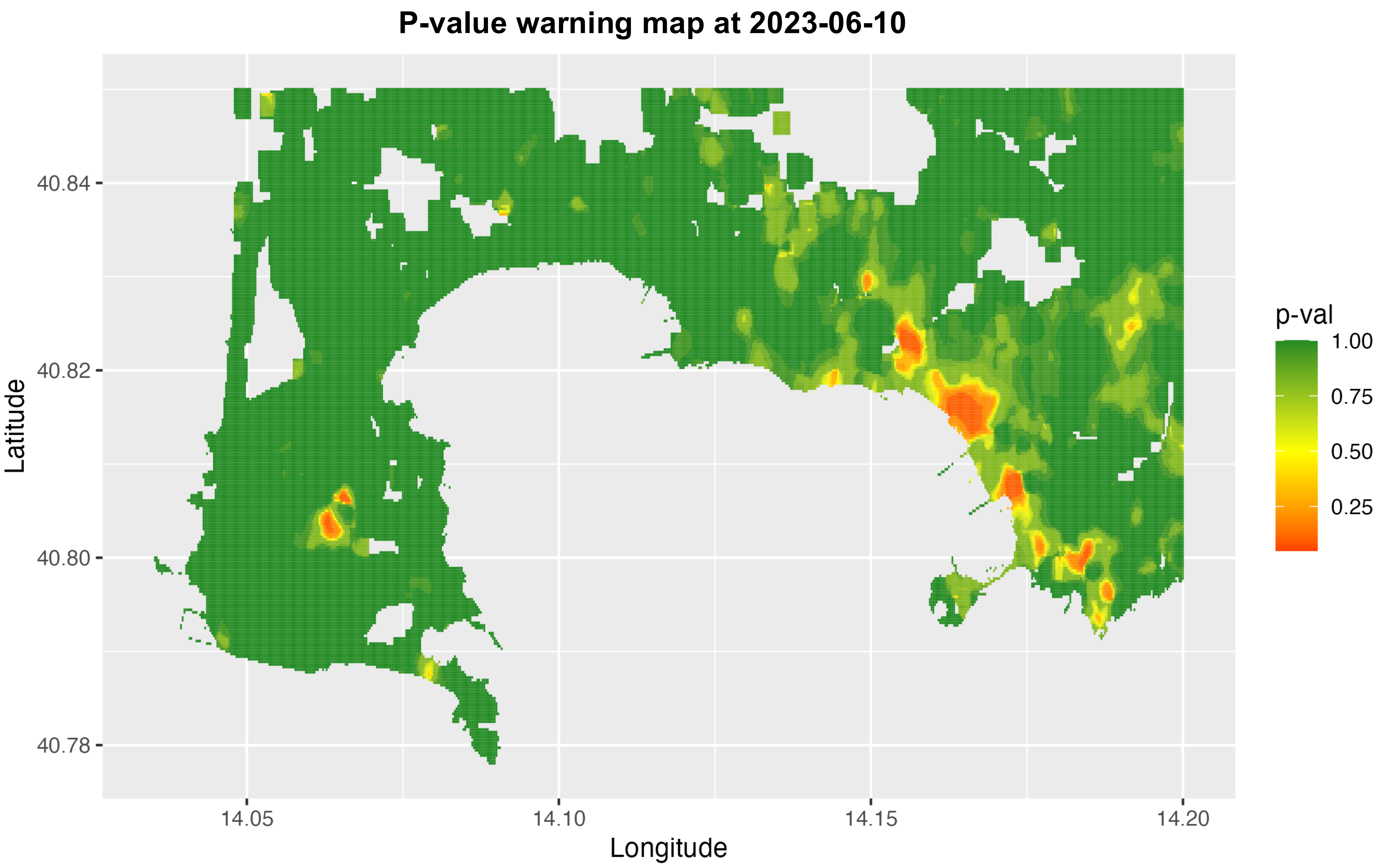
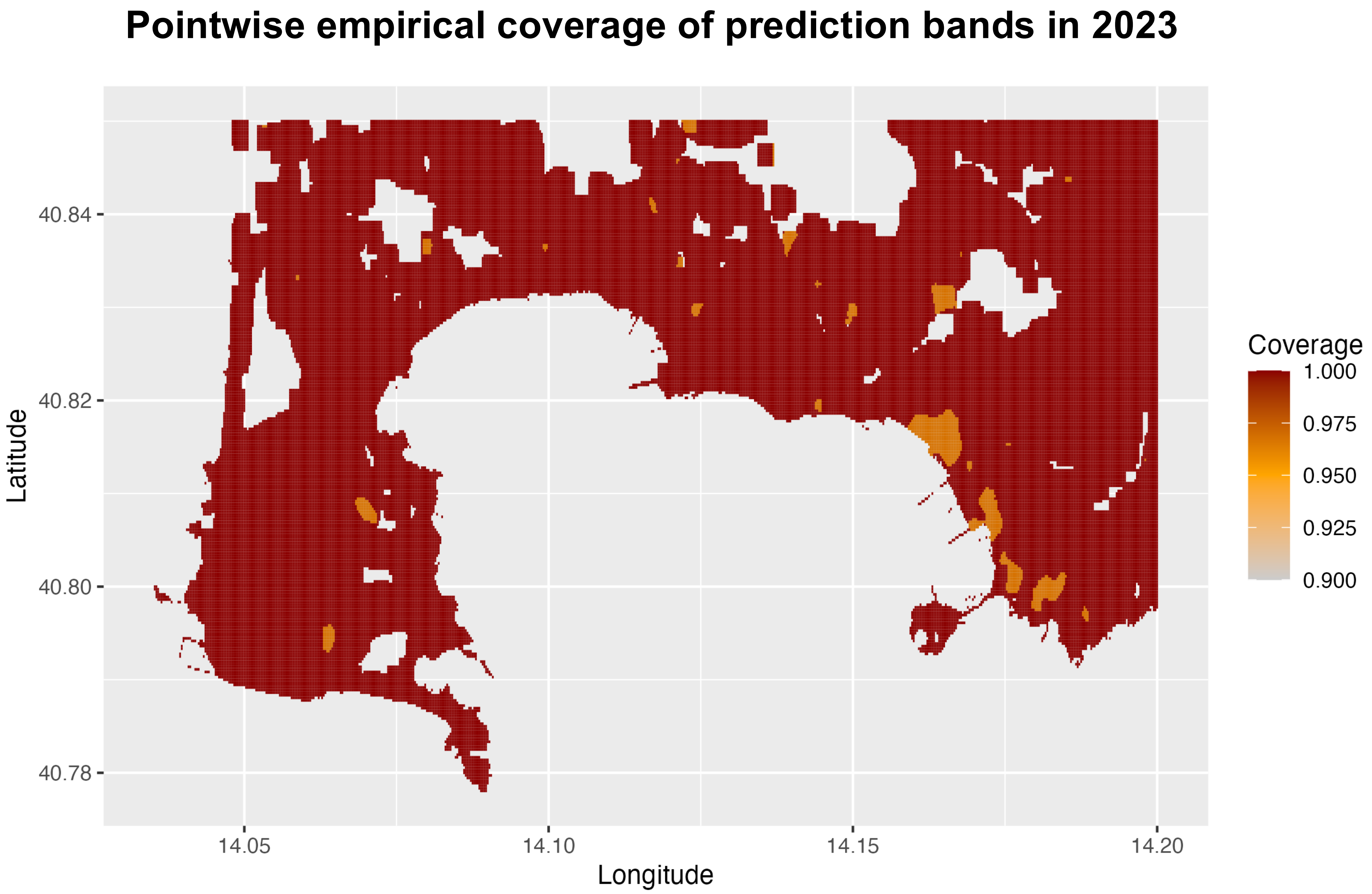
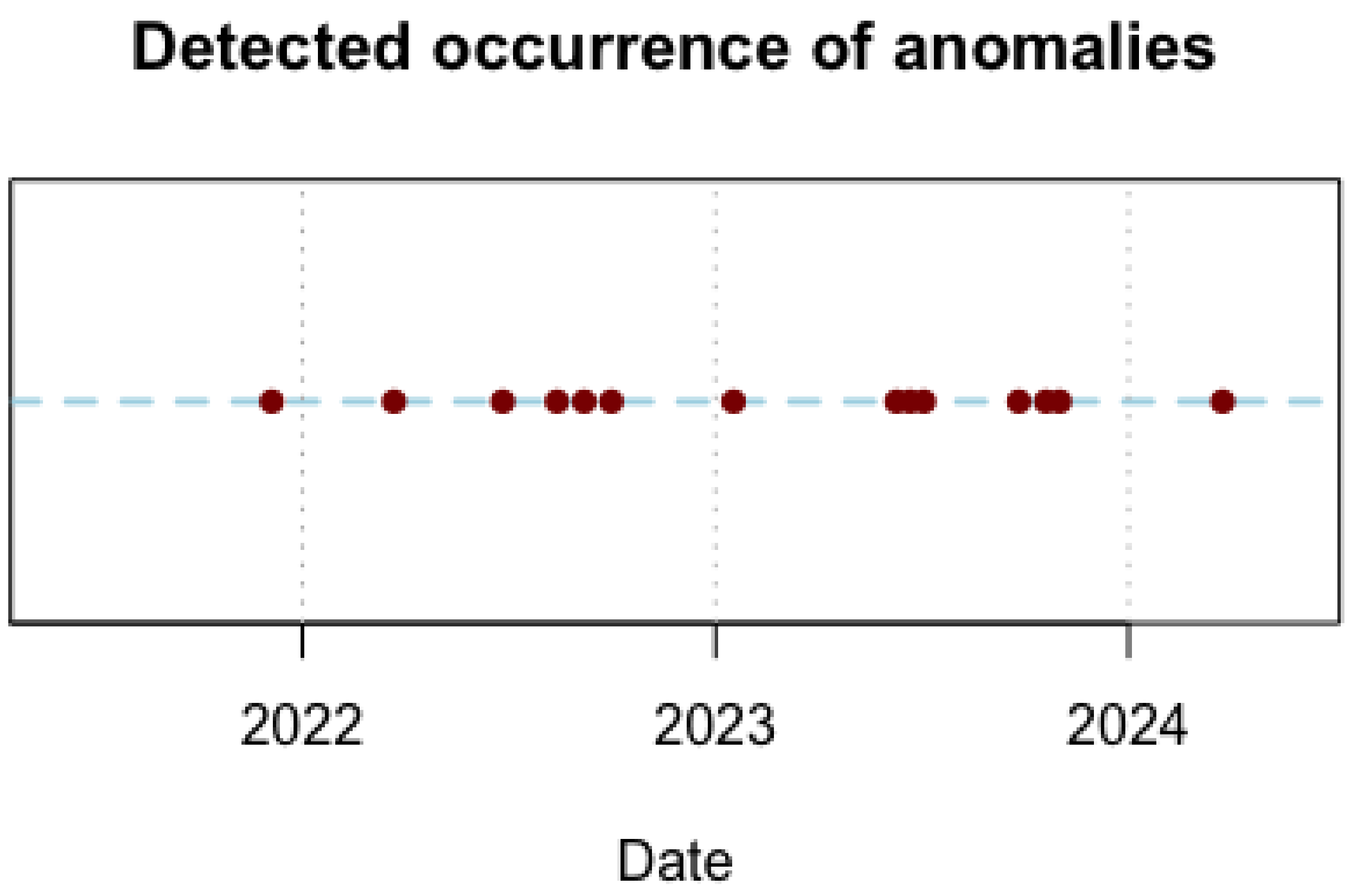


P-value warning map for June 10, 2023.

Continuous monitoring over time



Empirical coverage map at 10%.



Timeline of detected anomalies.

Conclusions and future work

Key aspects of the proposed monitoring tool

- Assumption-free approach
- Integration of spatial and temporal dynamics of ground displacement
- Localization of the anomalies
- Controlled risk of false warnings

Future extensions of the monitoring tool

- Joint monitoring of higher order derivatives
- Adaptation of the tool to account for noisy ground displacement observations
- Filling of the information in incoherent pixels

Thank you!

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[2] P. Berardino, G. Fornaro, R. Lanari, E. Sansosti, A new algorithm for surface deformation monitoring based on small baseline differential SAR interferograms. IEEE Trans. Geosci. Remote Sens. 40(11) (2002) 2375–2383.

[3] T. Bortolotti, F. Casu, M. Virelli, D. Tapete, M. Siciliani de Cumis, A. Menafoglio, S. Vantini (2024). Remote monitoring of ground motion exploiting functional data analysis and nonparametric prediction on multi-temporal DInSAR data. Congress Proceedings of the 75th International Astronautical Congress.

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