Component Separation Methods

Alessandro Carones

TOR VERGATA
UNIVERSITY OF ROME

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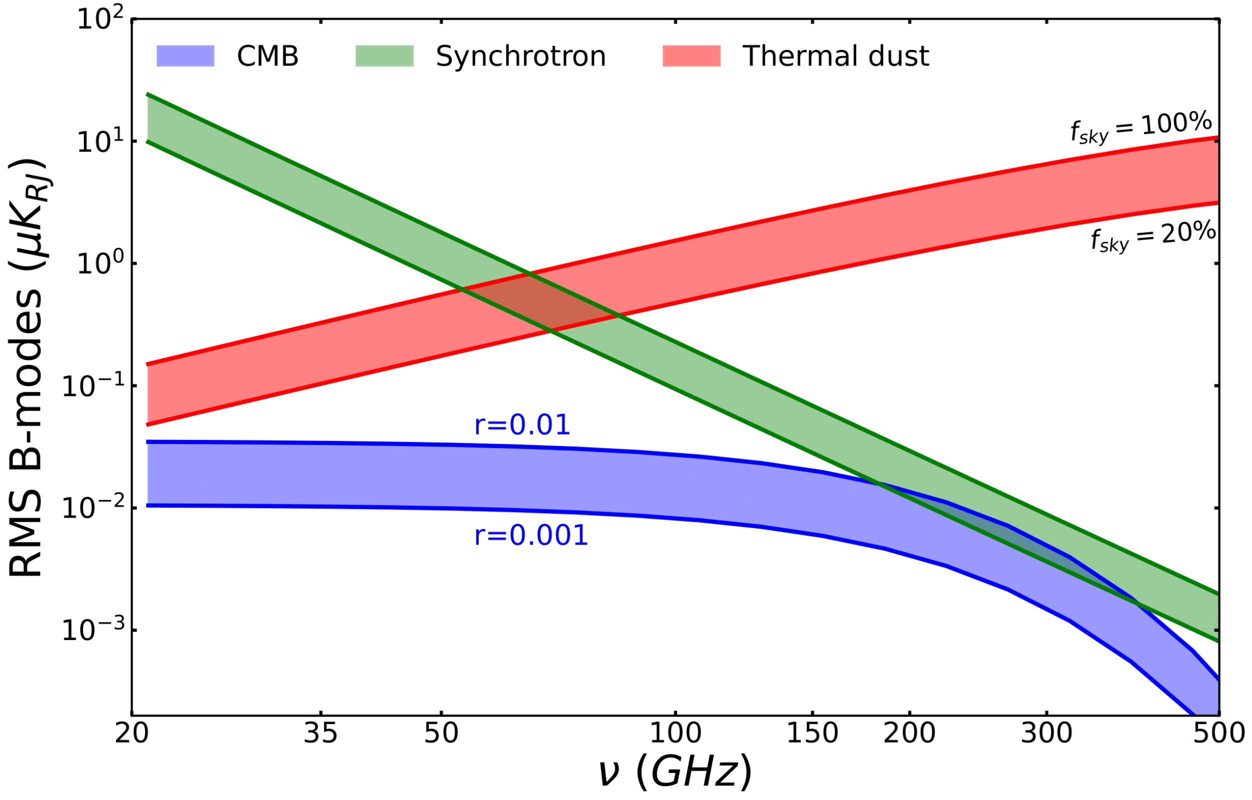
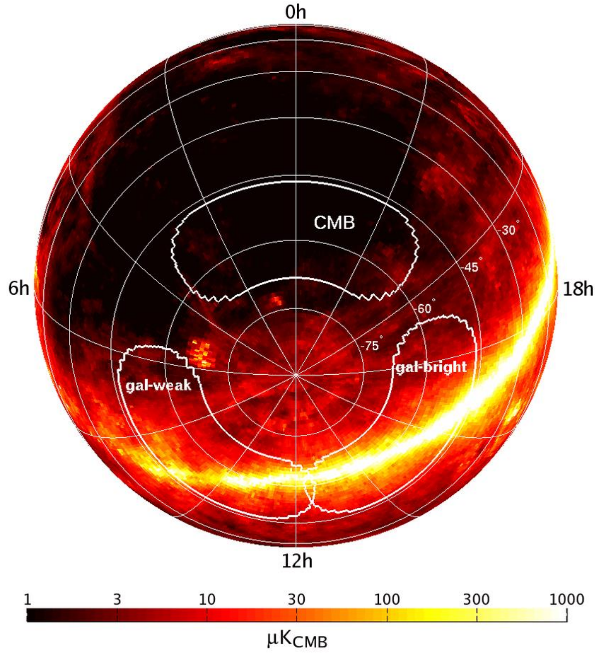
COMPONENT SEPARATION PROBLEM

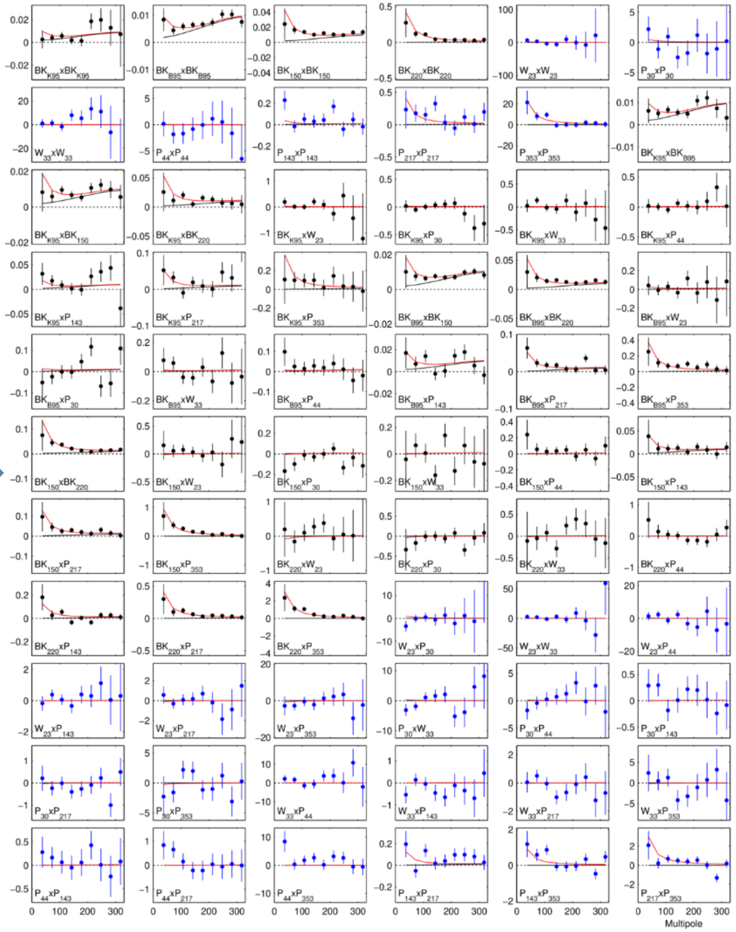


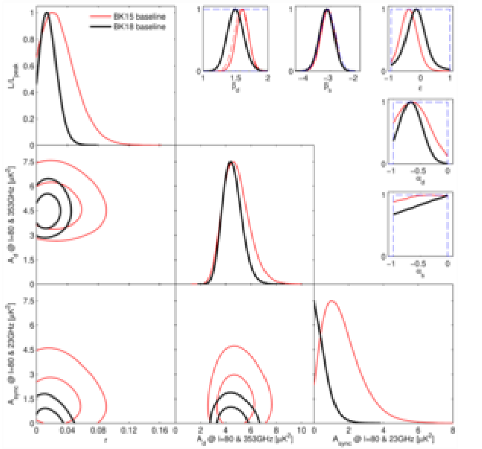
- All over the sky, B-mode foregrounds at their minimum are at a level $r\sim[0.05-1.5]$ (Krachmalnicoff et al., 2016)
- Future CMB experiments need effective component separation methods to achieve the targeted sensitivity

POWER SPECTRUM BASED – SMALL PATCHES



(Chiang et al., 2010)

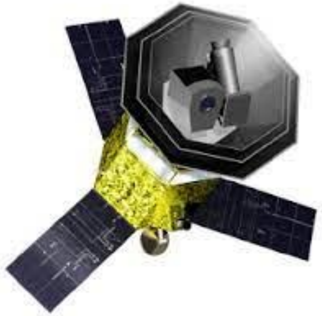




$$\mathcal{D}^{\nu_1\times\nu_2}_{\ell,BB} = A_{\rm d}\Delta'_{\rm d} f_{\rm d}^{\nu_1} f_{\rm d}^{\nu_2}\left(\frac{\ell}{80}\right)^{\alpha_{\rm d}} + A_{\rm sync}\Delta'_{\rm s} f_{\rm s}^{\nu_1} f_{\rm s}^{\nu_2}\left(\frac{\ell}{80}\right)^{\alpha_{\rm s}} + \epsilon\sqrt{A_{\rm d}A_{\rm sync}}\left(f_{\rm d}^{\nu_1} f_{\rm s}^{\nu_2} + f_{\rm s}^{\nu_1} f_{\rm d}^{\nu_2}\right)\left(\frac{\ell}{80}\right)^{(\alpha_{\rm d}+\alpha_{\rm s})/2}$$

(BICEP/Keck Collab., 2021)

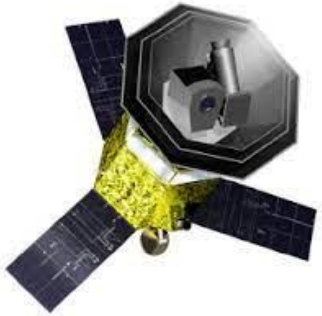
MAP-BASED COMPONENT SEPARATION



$$d = \begin{pmatrix} A_{CMB} & & \\ & A_d & \\ & & A_s \end{pmatrix} \begin{pmatrix} s_{CMB} \\ s_{dust} \\ s_{synch} \end{pmatrix} + n$$

Mixing matrix (SEDs) | Sky signals | Noise

MAP-BASED COMPONENT SEPARATION



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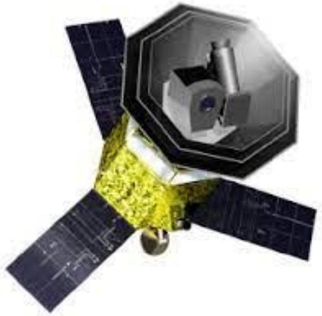
Mixing matrix (SEDs) — Sky signals — Noise

Minimum variance

$$\mathbb{E}\left[\left(\sum_i w_i d_i\right)^2\right]$$

Global variance minimum

MAP-BASED COMPONENT SEPARATION



$$\boldsymbol{d} = \begin{pmatrix} A_{CMB} & & \\ & A_d & \\ & & A_s \end{pmatrix} \begin{pmatrix} s_{CMB} \\ s_{dust} \\ s_{synch} \end{pmatrix} + \boldsymbol{n}$$

Mixing matrix (SEDs)

Sky signals

Noise

Minimum variance

$$\mathbb{E}\left[\left(\sum_i w_i d_i\right)^2\right]$$

Global variance minimum

Parametric

$$\chi^2(\hat{\gamma}) = \sum_i \left(\frac{d_i(\hat{\gamma}) - m_i(\hat{\gamma})}{\sigma_i(\hat{\gamma})}\right)^2$$

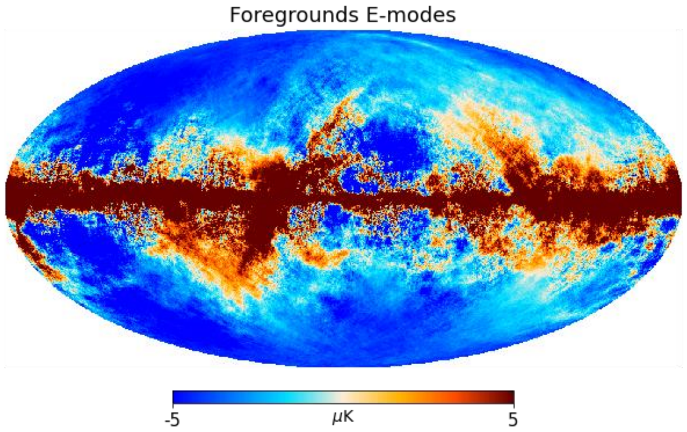
Global chi-square minimum

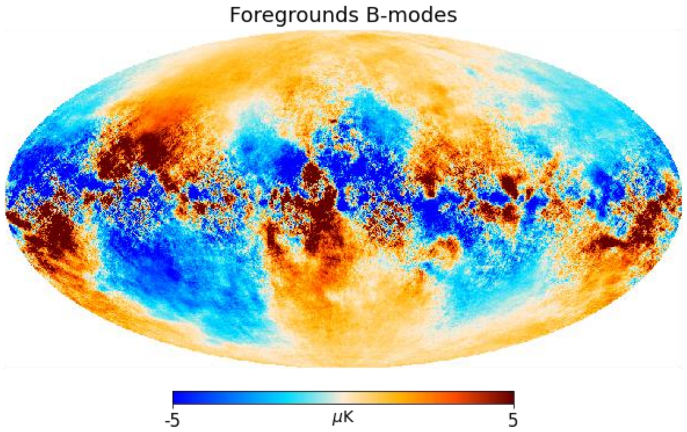
1. FIELD OF APPLICATION

Minimum variance

$$E_{\ell m} = -\frac{1}{2}\left(a_{2,\ell m} + a_{-2,\ell m}\right)$$

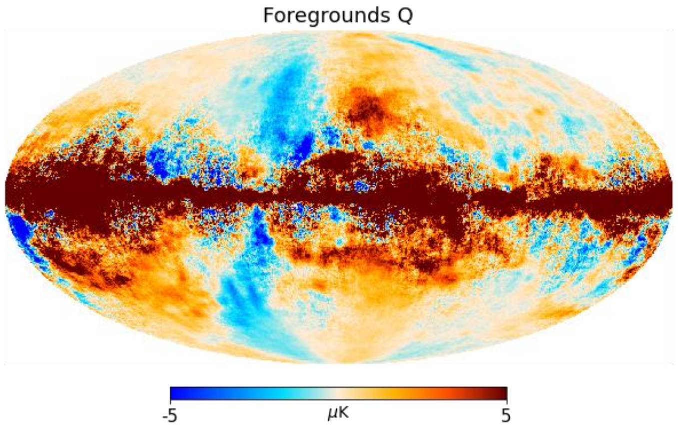
$$B_{\ell m} = -\frac{1}{2i}\left(a_{2,\ell m} - a_{-2,\ell m}\right)$$

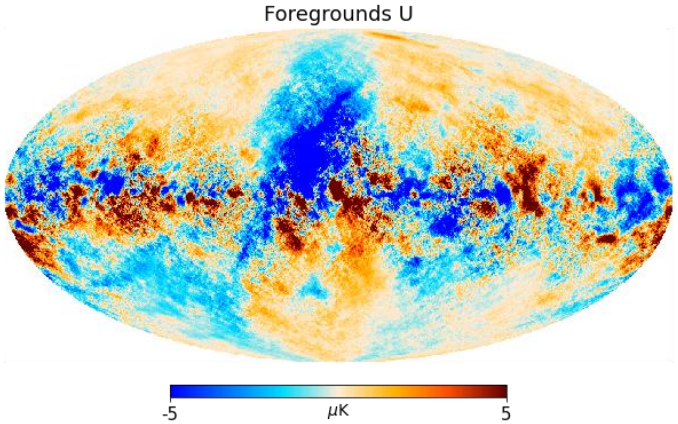




Parametric

$$(Q \pm iU)(\hat{\gamma}) = \sum_{\ell m} a_{\pm 2,\ell m}\ {}_{\pm 2}Y_{\ell m}(\hat{\gamma})$$





2. AVAILABLE ALGORITHMS

Minimum variance

- Internal Linear Combination (ILC) (Bennett et al., 2003)
- Harmonic ILC (Tegmark et al., 2003)
- SMICA (Delabrouille et al., 2003)
- Needlet ILC (NILC) (Delabrouille et al., 2009)

Component separation golden year (2015)

- Polarization ILC (Fernández-Cobos et al., 2016)
- constrained Moments ILC (cMILC) (Remazeilles et al., 2021)
- Multi-Clustering NILC (MC-NILC) (Carones et al., 2023a)
- Optimised cMILC (Carones et al., in prep.)

Parametric

- Commander (Eriksen et al., 2008)
- FGBuster (Stompor et al., 2009)

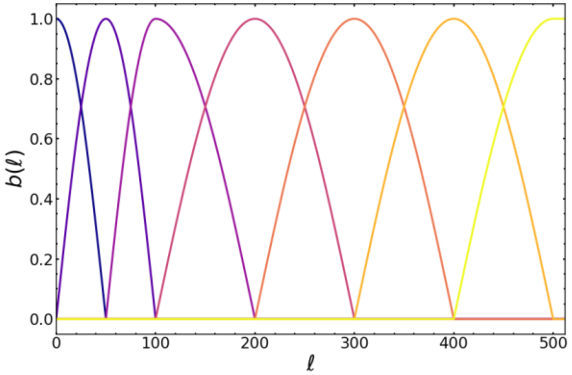
- Commander3 (Galloway et al., 2022)
- FGBuster (Errard & Poletti, in prep)
- FGCluster (Puglisi et al., 2022)
- B-SeCRET (de la Hoz, 2020)
- Moments fitting (Mangilli et al., 2021; Vacher et al, 2022)

3. IMPLEMENTATION

Minimum variance

- Needlet ILC (NILC):

$$\beta_j^i(\hat{\gamma}) = \sum_{\ell,m} \left(B^i_{\ell m} \cdot b_j(\ell)\right) \cdot Y_{\ell m}(\hat{\gamma})$$



$$\beta_j^{NILC}(\hat{\gamma}) = \sum_{i=1}^{N_\nu} \omega_i^j(\hat{\gamma}) \cdot \beta_j^i(\hat{\gamma}) = \sum_{\ell,m} a^{NILC}_{\ell m,j} \cdot Y_{\ell m}(\hat{\gamma})$$

with $\begin{cases} \sum_{i=1}^{N_\nu} \omega_i^j(\hat{\gamma}) \cdot A^i_{\text{CMB}} = 1 \\ \langle (\beta_j^{NILC})^2 \rangle \text{ minimum} \end{cases}$

$$\omega^j(\hat{\gamma}) = \frac{A^T_{\text{CMB}} C_j(\hat{\gamma})^{-1}}{A^T_{\text{CMB}} C_j(\hat{\gamma})^{-1} A_{\text{CMB}}} \qquad C_j^{ik} = \langle \beta_j^i \beta_j^k \rangle$$

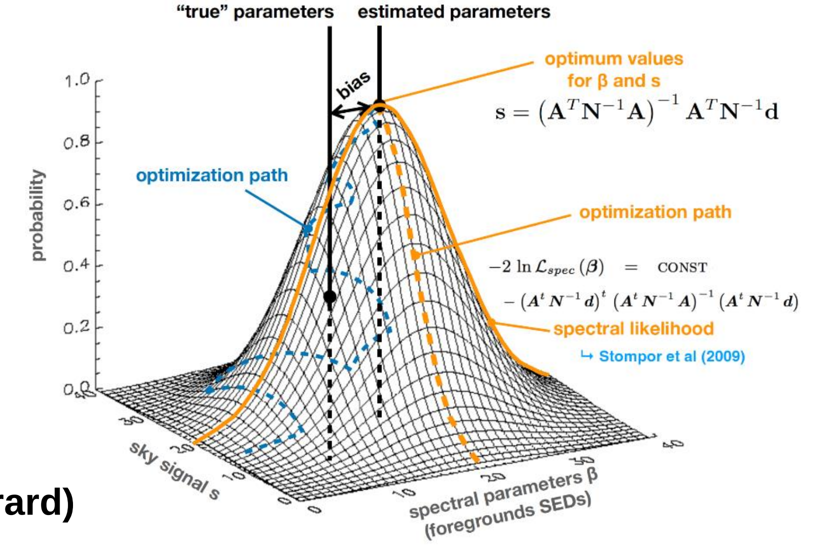
Parametric

- Commander:

$$d_{j,t} = g_{j,t} \mathsf{P}_{tp,j} \left[\mathsf{B}^{\text{symm}}_{pp',j} \sum_c \mathsf{M}_{cj}(\beta_{p'}, \Delta^j_{\text{bp}}) a^c_{p'} + \mathsf{B}^{\text{asymm}}_{pp',j} \left(s^{\text{orb}}_{j,t} + s^{\text{fsl}}_{j,t} \right) \right] + $$
$$+ s^{1\,\text{Hz}}_{j,t} + n^{\text{corr}}_{j,t} + n^{\text{w}}_{j,t}.$$

$$\begin{aligned}
\boldsymbol{g} &\leftarrow P(\boldsymbol{g} \mid \boldsymbol{d}, \quad \xi_n, \Delta_{\text{bp}}, \boldsymbol{a}, \beta, C_\ell) \\
\boldsymbol{n}_{\text{corr}} &\leftarrow P(\boldsymbol{n}_{\text{corr}} \mid \boldsymbol{d}, \boldsymbol{g}, \quad \xi_n, \Delta_{\text{bp}}, \boldsymbol{a}, \beta, C_\ell) \\
\xi_n &\leftarrow P(\xi_n \mid \boldsymbol{d}, \boldsymbol{g}, \boldsymbol{n}_{\text{corr}}, \quad \Delta_{\text{bp}}, \boldsymbol{a}, \beta, C_\ell) \\
\Delta_{\text{bp}} &\leftarrow P(\Delta_{\text{bp}} \mid \boldsymbol{d}, \boldsymbol{g}, \boldsymbol{n}_{\text{corr}}, \xi_n, \quad \boldsymbol{a}, \beta, C_\ell) \\
\beta &\leftarrow P(\beta \mid \boldsymbol{d}, \boldsymbol{g}, \boldsymbol{n}_{\text{corr}}, \xi_n, \Delta_{\text{bp}}, \quad C_\ell) \\
\boldsymbol{a} &\leftarrow P(\boldsymbol{a} \mid \boldsymbol{d}, \boldsymbol{g}, \boldsymbol{n}_{\text{corr}}, \xi_n, \Delta_{\text{bp}}, \quad \beta, C_\ell) \\
C_\ell &\leftarrow P(C_\ell \mid \boldsymbol{d}, \boldsymbol{g}, \boldsymbol{n}_{\text{corr}}, \xi_n, \Delta_{\text{bp}}, \boldsymbol{a}, \beta \quad)
\end{aligned}$$

- FGBuster:



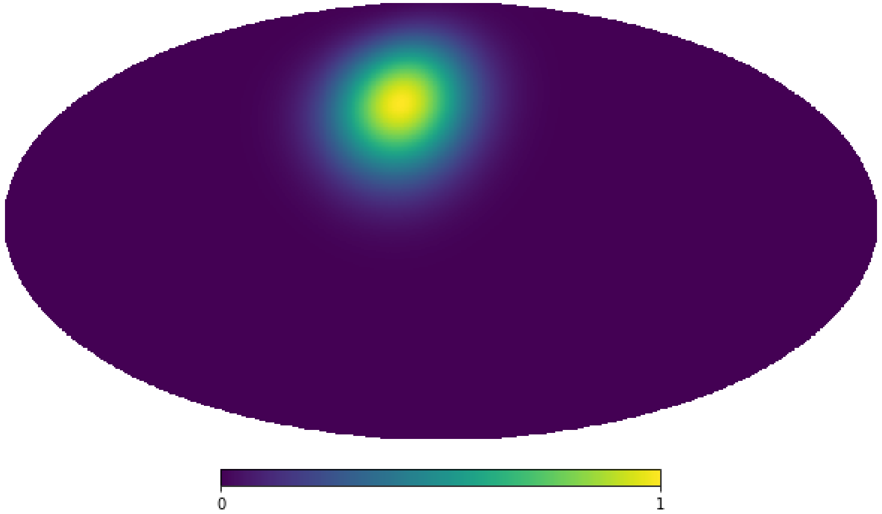
(Credit: J.Errard)

3.1 IMPLEMENTATION

Minimum variance

- Needlet ILC (NILC):

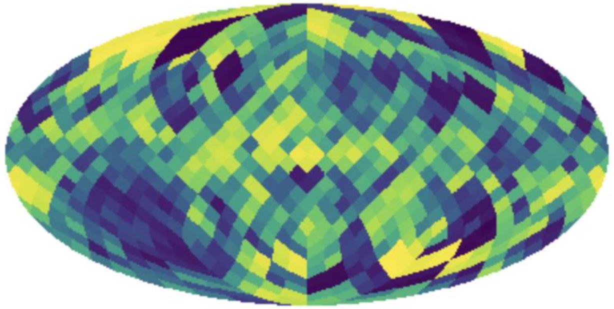
$$C_j^{ik} = \langle \beta_j^i \beta_j^k \rangle$$



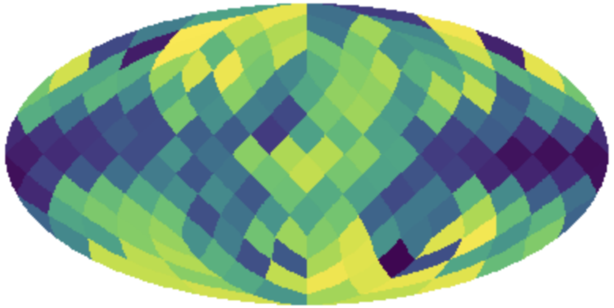
Parametric

- FGBuster:

$\boldsymbol{\beta_d}$



$\boldsymbol{T_d}$



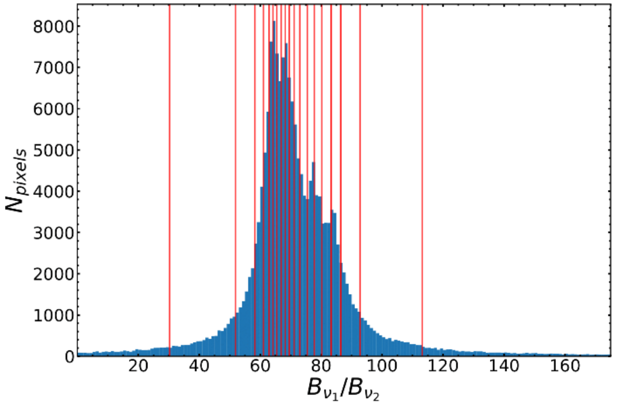
$\boldsymbol{\beta_s}$

4. INTERFACE WITH DATA-DRIVEN DOMAINS

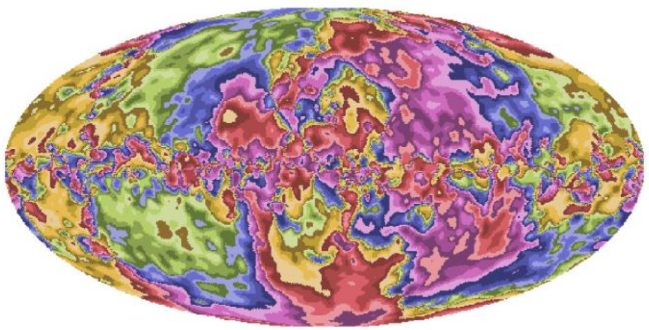
Minimum variance

- Multi-Clustering NILC (MC-NILC):

$$\frac{B^{hf}_{fgds}}{B^{119}_{fgds}} = \frac{B^{hf}_{dust} + B^{hf}_{synch}}{B^{119}_{dust} + B^{119}_{synch}} = \frac{B^{hf}_{dust}}{B^{119}_{dust}} \cdot \frac{1 + \frac{B^{hf}_{synch}}{B^{hf}_{dust}}}{1 + \frac{B^{119}_{synch}}{B^{119}_{dust}}} \simeq \frac{B^{hf}_{dust}}{B^{119}_{dust}} \cdot \frac{1}{1 + \frac{B^{119}_{synch}}{B^{119}_{dust}}}$$



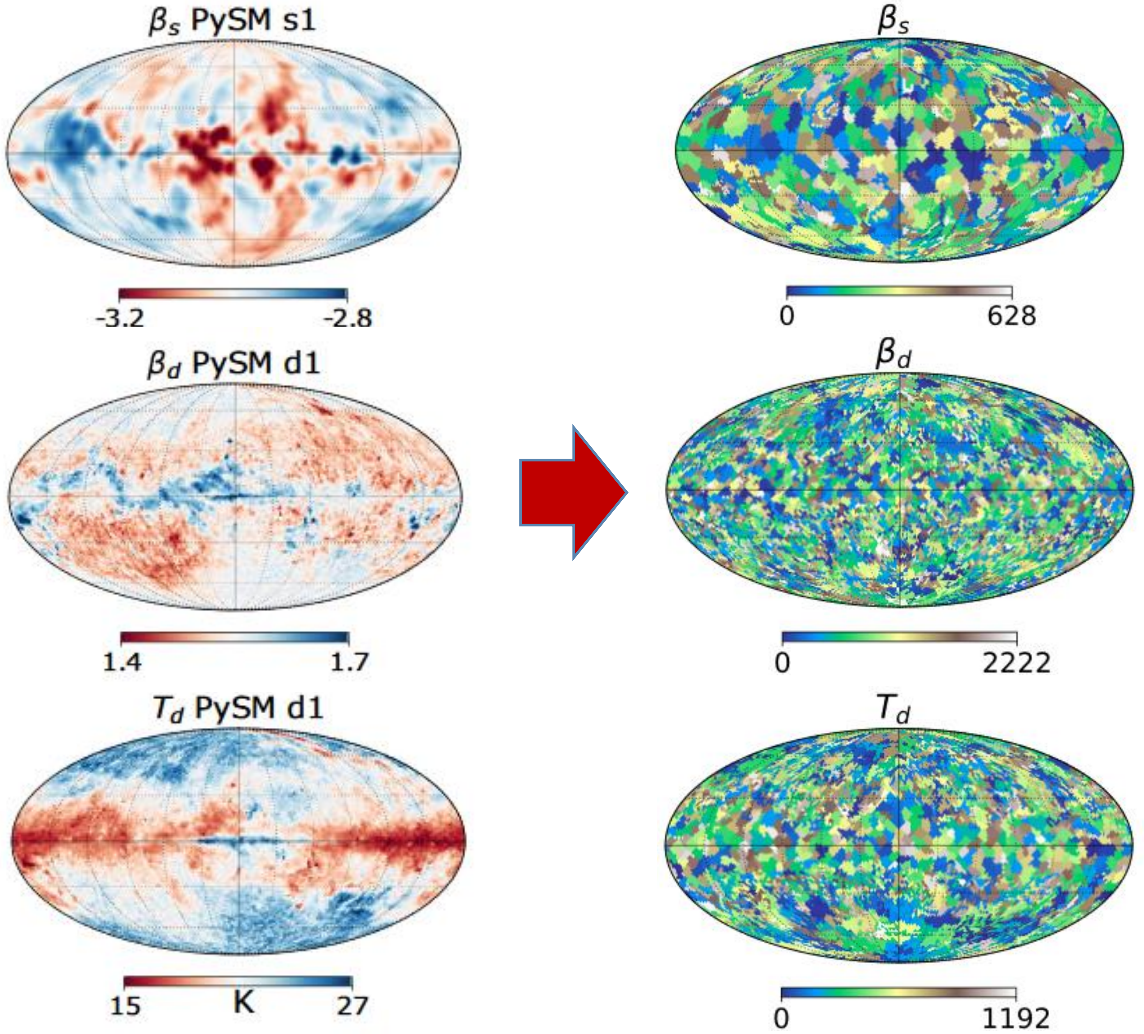
(Carones et al., 2023a)



Parametric

(Puglisi et al., 2022)

- FGCluster:



5. INTERFACE WITH MOMENTS EXPANSION

(Credit: M.Remazeilles)

Synchrotron

$$
\begin{aligned}
I_s(\nu,p) = {} & A_{\nu_s}(p) f_{\rm sync}\left(\nu,\bar{\beta}_s\right) \\
& + A_{\nu_s}(p)\left(\beta_s(p)-\bar{\beta}_s\right)\partial_{\beta_s} f_{\rm sync}\left(\nu,\bar{\beta}_s\right) \\
& + \frac{1}{2} A_{\nu_s}(p)\left(\beta_s(p)-\bar{\beta}_s\right)^2 \partial^2_{\beta_s} f_{\rm sync}\left(\nu,\bar{\beta}_s\right) \\
& + \mathcal{O}\left(\beta_s^3\right),
\end{aligned}
$$

Thermal dust

$$
\begin{aligned}
I_d(\nu,p) = {} & A_{\nu_d}(p) f_{\rm dust}\left(\nu,\bar{\beta}_d\right) \\
& + A_{\nu_d}(p)\left(\beta_d(p)-\bar{\beta}_d\right)\partial_{\beta_d} f_{\rm dust}\left(\nu,\bar{\beta}_d,\bar{T}_d\right) \\
& + A_{\nu_d}(p)\left(T_d(p)-\bar{T}_d\right)\partial_{T_d} f_{\rm dust}\left(\nu,\bar{\beta}_d,\bar{T}_d\right) \\
& + \frac{1}{2} A_{\nu_d}(p)\left(\beta_d(p)-\bar{\beta}_d\right)^2 \partial^2_{\beta_d} f_{\rm dust}\left(\nu,\bar{\beta}_d,\bar{T}_d\right) \\
& + \frac{1}{2} A_{\nu_d}(p)\left(T_d(p)-\bar{T}_d\right)^2 \partial^2_{T_d} f_{\rm dust}\left(\nu,\bar{\beta}_d,\bar{T}_d\right) \\
& + A_{\nu_d}(p)\left(\beta_d(p)-\bar{\beta}_d\right)\left(T_d(p)-\bar{T}_d\right)\partial_{\beta_d}\partial_{T_d} f_{\rm dust}\left(\nu,\bar{\beta}_d,\bar{T}_d\right) \\
& + \mathcal{O}\left(\beta_d^3, T_d^3\right),
\end{aligned}
$$

(Chluba et al., 2017)

Minimum variance

- cMILC:

$$
\begin{cases}
\sum_\nu w(\nu)\cdot f_{\rm CMB}(\nu) = 1 \\
\sum_\nu w(\nu)\cdot f_{sync}(\nu) = 0 \\
\sum_\nu w(\nu)\cdot f_{dust}(\nu) = 0 \\
\sum_\nu w(\nu)\cdot \frac{\partial f_{\rm sync}}{\partial \bar{\beta}_s}(\nu) = 0 \\
\sum_\nu w(\nu)\cdot \frac{\partial f_{\rm dust}}{\partial \bar{\beta}_d}(\nu) = 0 \\
\sum_\nu w(\nu)\cdot \frac{\partial f_{dust}}{\partial \bar{T}_d}(\nu) = 0 \\
\dots
\end{cases}
$$

$$w^{\rm T} = e^{\rm T}\left(\mathrm{A}^{\rm T}\mathrm{C}^{-1}\mathrm{A}\right)^{-1}\mathrm{A}^{\rm T}\mathrm{C}^{-1}$$

$$\mathrm{A} = \left(f_{CMB}\quad f_{sync}\quad f_{dust}\quad \cdots \quad \partial_{\bar{T}_d} f_{dust}\right)$$

$$e^{\rm T} = (1\ 0\ 0\ \dots 0)$$

(Remazeilles et al., 2021)

Parametric

- Moments fitting:

$$
\begin{aligned}
\mathcal{D}_\ell(\nu_i\times\nu_j) = {} & \frac{I_{\nu_i}(\beta_0(\ell),T_0(\ell))I_{\nu_j}(\beta_0(\ell),T_0(\ell))}{I_{\nu_0}(\beta_0(\ell),T_0(\ell))^2}\cdot\Bigg\{ \\
0^{\rm th}\text{ order} & \left\{\ \mathcal{D}_\ell^{A\times A}\right. \\
1^{\rm st}\text{ order }\beta & \left\{\begin{array}{l} +\mathcal{D}_\ell^{A\times\omega_1^\beta}\left[\ln\left(\frac{\nu_i}{\nu_0}\right)+\ln\left(\frac{\nu_j}{\nu_0}\right)\right] \\ +\mathcal{D}_\ell^{\omega_1^\beta\times\omega_1^\beta}\left[\ln\left(\frac{\nu_i}{\nu_0}\right)\ln\left(\frac{\nu_j}{\nu_0}\right)\right]\end{array}\right. \\
1^{\rm st}\text{ order }T & \left\{\begin{array}{l} +\mathcal{D}_\ell^{A\times\omega_1^T}\left(\Theta_i+\Theta_j-2\Theta_0\right) \\ +\mathcal{D}_\ell^{\omega_1^T\times\omega_1^T}\left(\Theta_i-\Theta_0\right)\left(\Theta_j-\Theta_0\right)\end{array}\right. \\
1^{\rm st}\text{ order }T\beta & \left\{\ +\mathcal{D}_\ell^{\omega_1^\beta\times\omega_1^T}\left[\ln\left(\frac{\nu_j}{\nu_0}\right)\left(\Theta_i-\Theta_0\right)+\ln\left(\frac{\nu_i}{\nu_0}\right)\left(\Theta_j-\Theta_0\right)\right]\right. \\
2^{\rm nd}\text{ order }\beta & \left\{\begin{array}{l} +\frac{1}{2}\mathcal{D}_\ell^{A\times\omega_2^\beta}\left[\ln^2\left(\frac{\nu_i}{\nu_0}\right)+\ln^2\left(\frac{\nu_j}{\nu_0}\right)\right] \\ +\frac{1}{2}\mathcal{D}_\ell^{\omega_1^\beta\times\omega_2^\beta}\left[\ln\left(\frac{\nu_i}{\nu_0}\right)\ln^2\left(\frac{\nu_j}{\nu_0}\right)+\ln\left(\frac{\nu_j}{\nu_0}\right)\ln^2\left(\frac{\nu_i}{\nu_0}\right)\right] \\ +\frac{1}{4}\mathcal{D}_\ell^{\omega_2^\beta\times\omega_2^\beta}\left[\ln^2\left(\frac{\nu_i}{\nu_0}\right)\ln^2\left(\frac{\nu_j}{\nu_0}\right)\right]\end{array}\right. \\
& +\dots\Bigg\},
\end{aligned}
$$

(Azzoni et al., 2020;

Mangilli et al., 2021;

Vacher et al, 2022)

6. IMPACT OF SYSTEMATICS

- Gain calibration
- Beams (Near and Far sidelobes)
- Bandpasses mismatch
- Polarization angle calibration
- Pointing
- HWP systematics
- …..

Minimum variance

In general, minimum variance methods well ‘absorb’ instrumental systematic effects, especially for low signal-to-noise analyses (Dick et al.,2010)

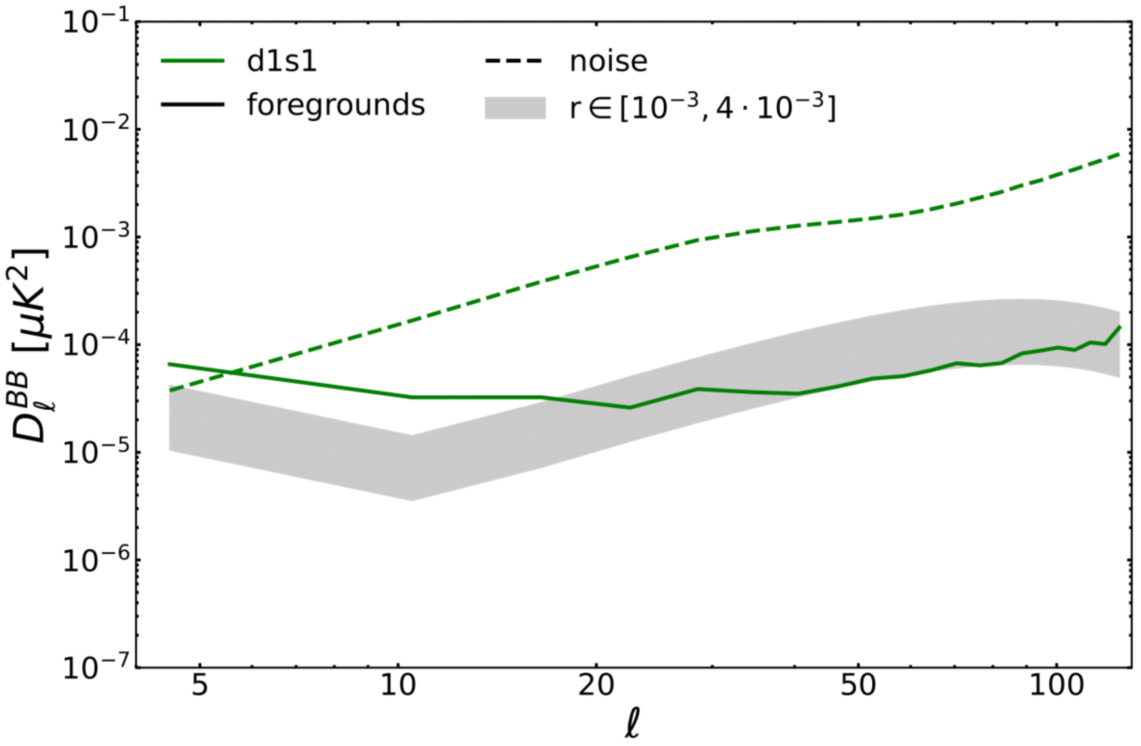
Parametric

- Systematics couple with foreground modeling
- Need to be incorporated within the fitting procedure

APPLICATION TO LITEBIRD ($\delta r \sim 0.001$)

Minimum variance

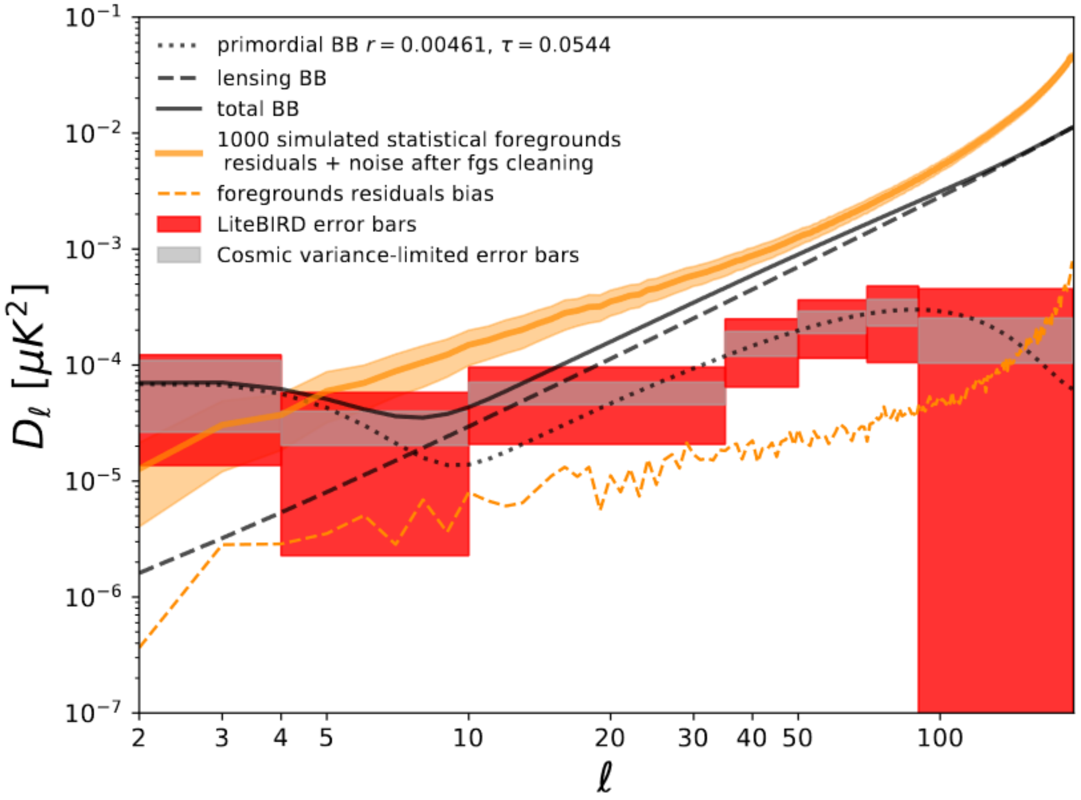
NILC, $f_{sky} = 50\%$



(Carones et al., 2023a)

Parametric

FGBuster, $f_{sky} = 50\%$

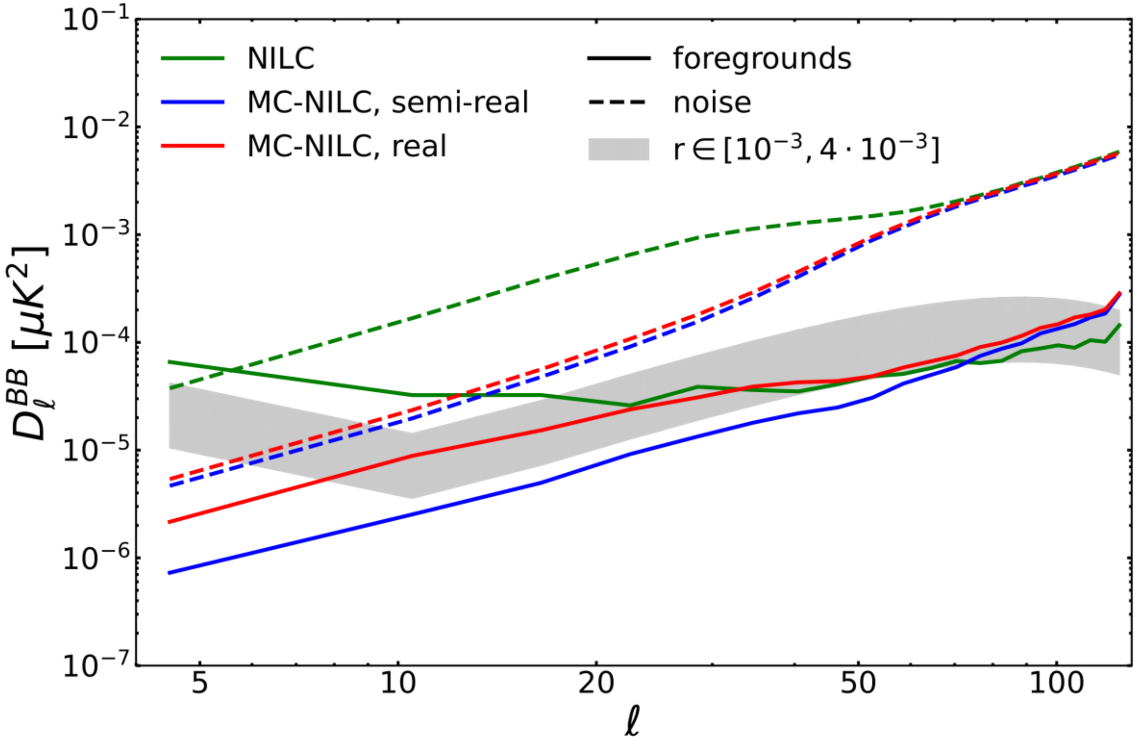


(LiteBIRD Collab., 2023)

APPLICATION TO LITEBIRD ($\delta r \sim 0.001$)

Minimum variance

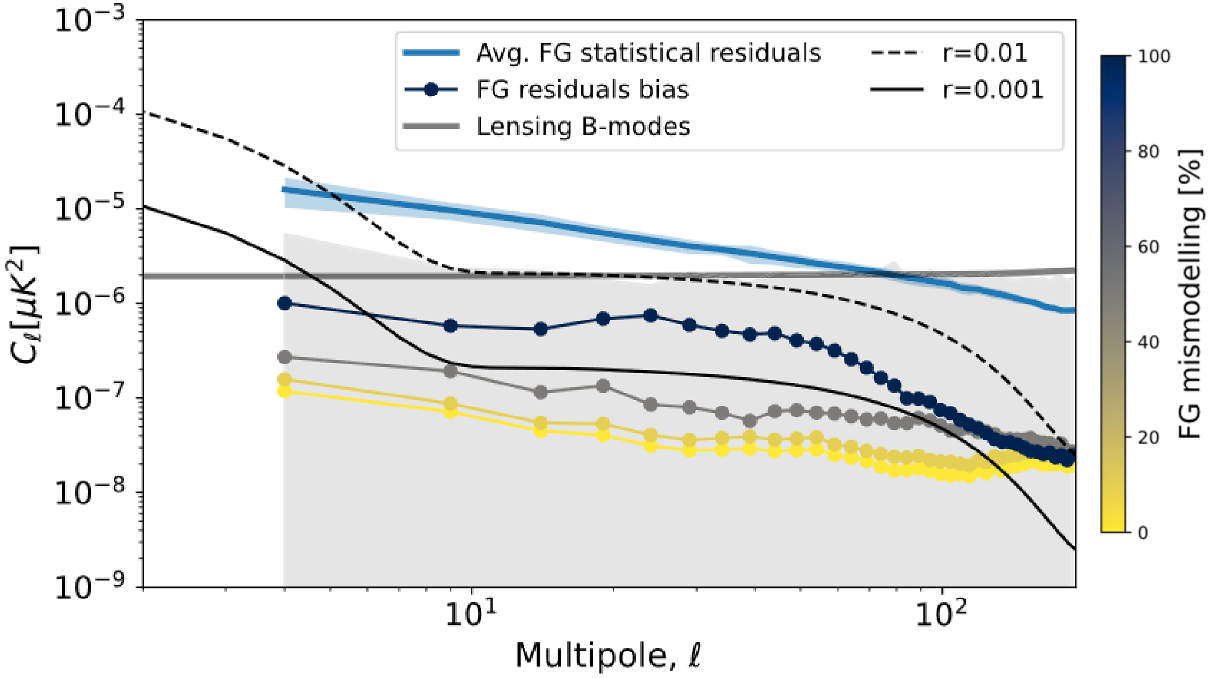
$f_{sky} = 50\%$



(Carones et al., 2023a)

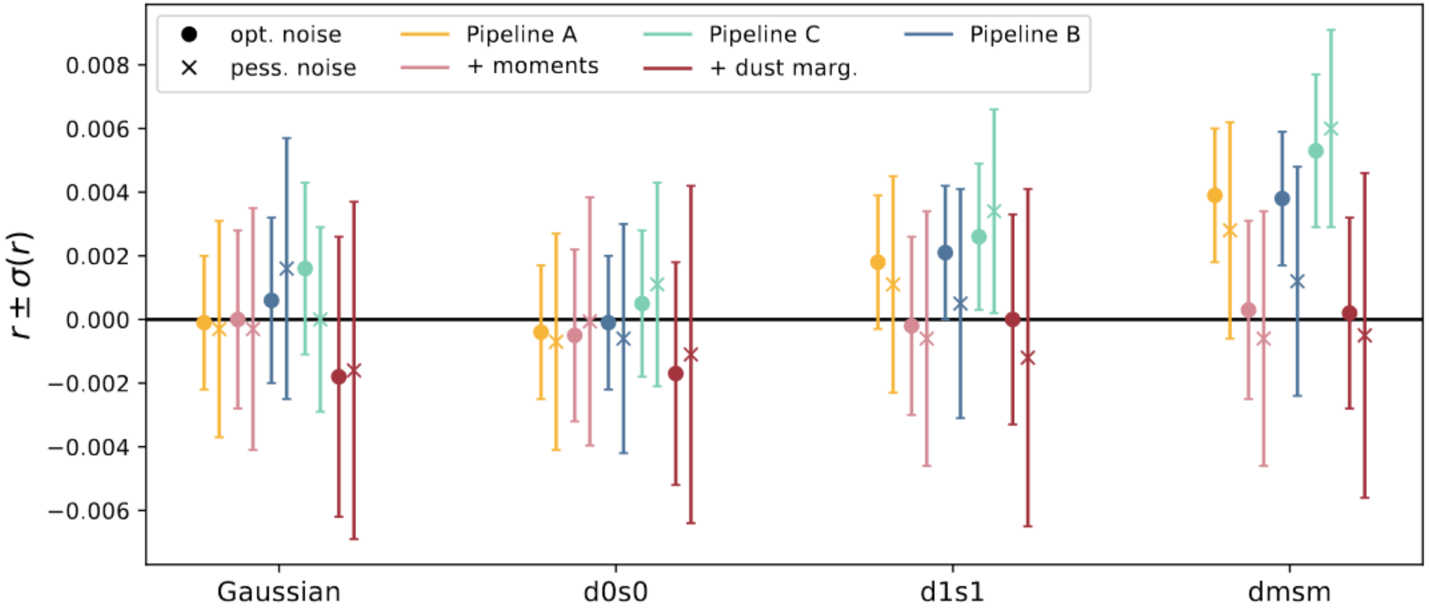
Parametric

FGCluster, $f_{sky} = 60\%$



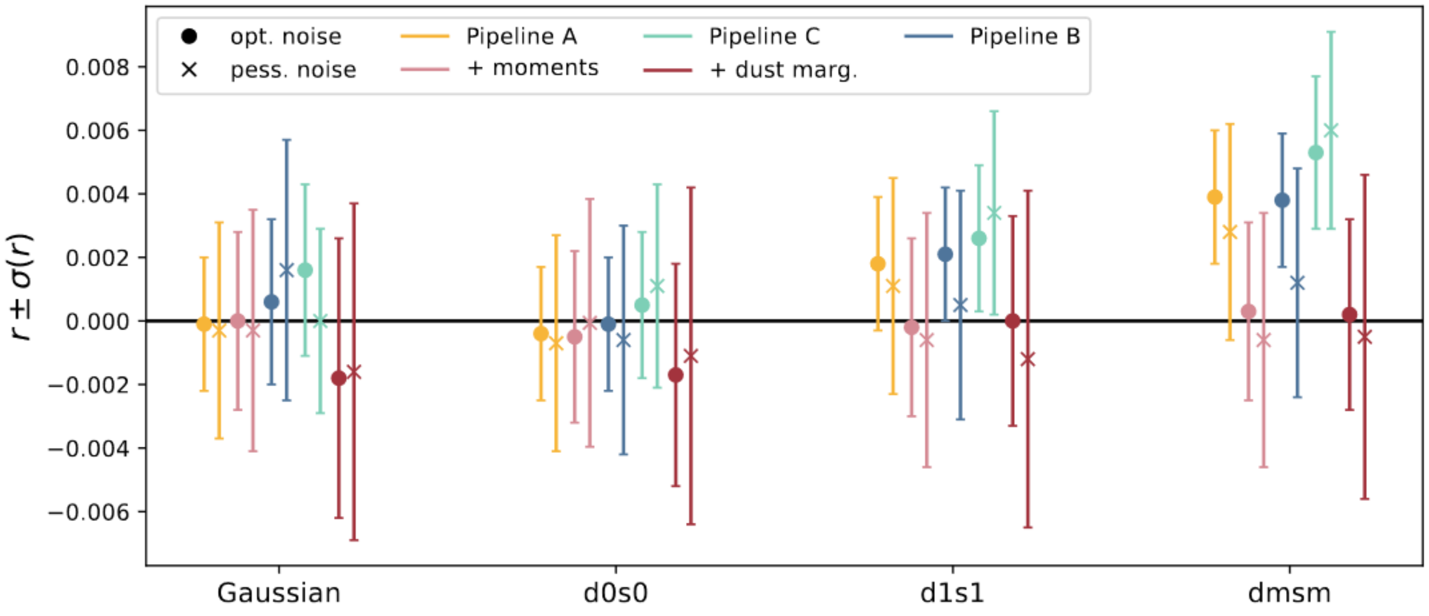
(Puglisi et al., 2022)

APPLICATION TO SO-SATs ($\delta r \sim 0.003$)



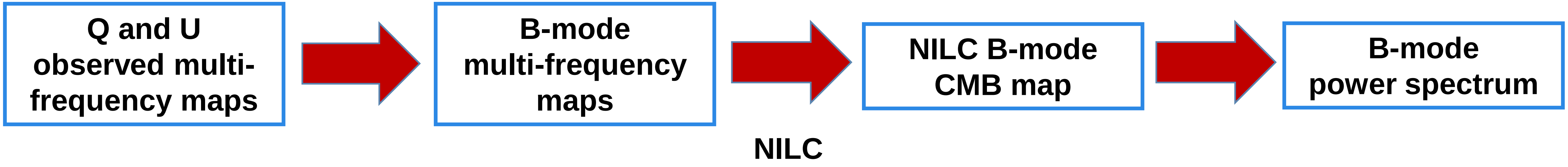
(Wolz et al., 2023)

APPLICATION TO SO-SATs ($\delta r \sim 0.003$)

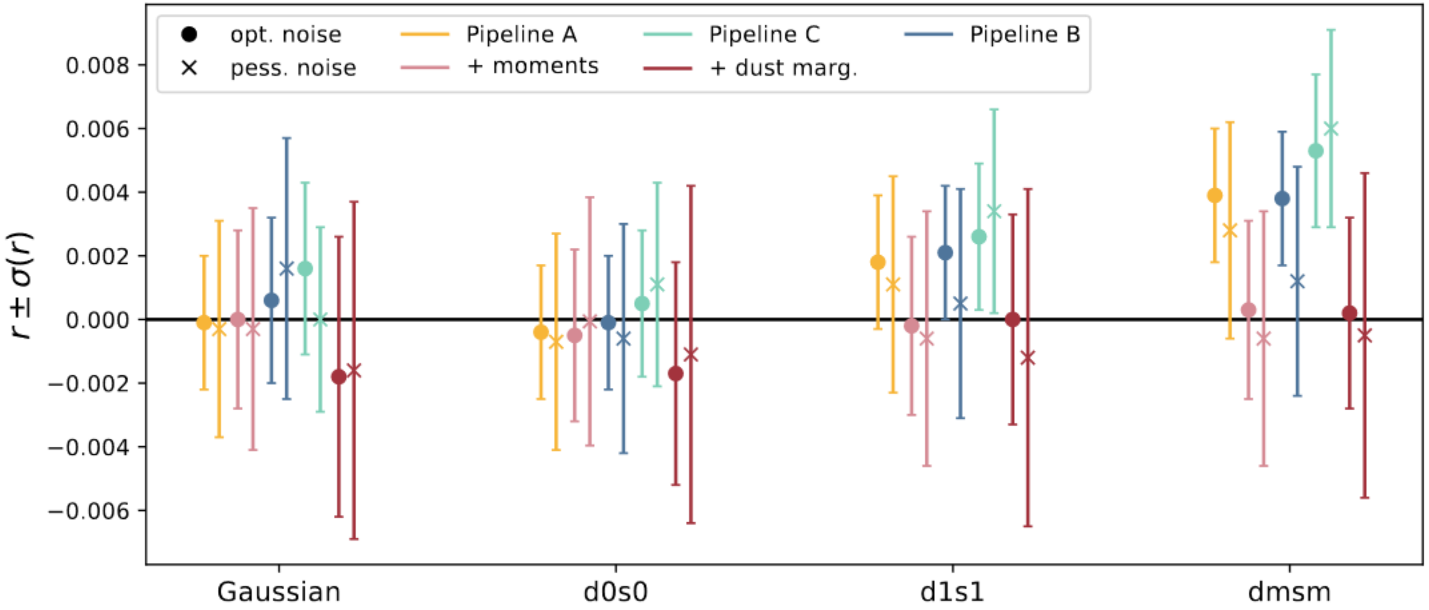


(Wolz et al., 2023)

Minimum variance



APPLICATION TO SO-SATs ($\delta r \sim 0.003$)

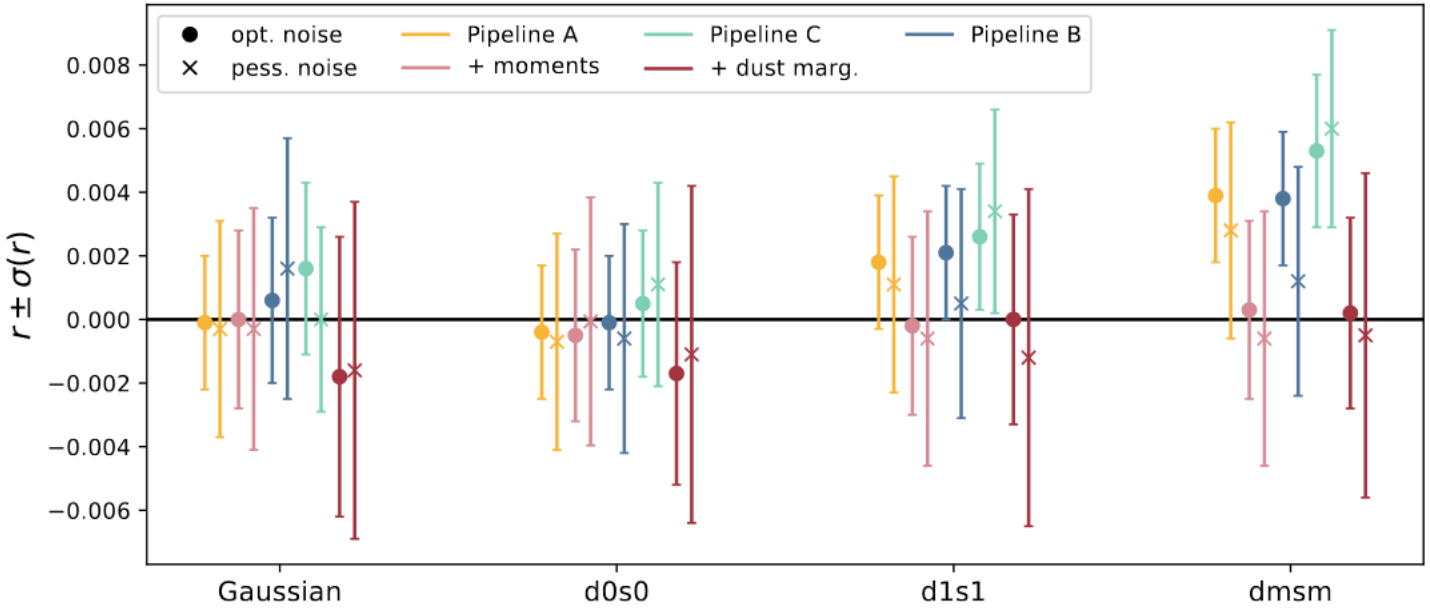


(Wolz et al., 2023)

Minimum variance

Q and U observed multi-frequency maps → (E-B leakage) → **B-mode multi-frequency maps** → (NILC) → **NILC B-mode CMB map** → **B-mode power spectrum**

APPLICATION TO SO-SATs ($\delta r \sim 0.003$)



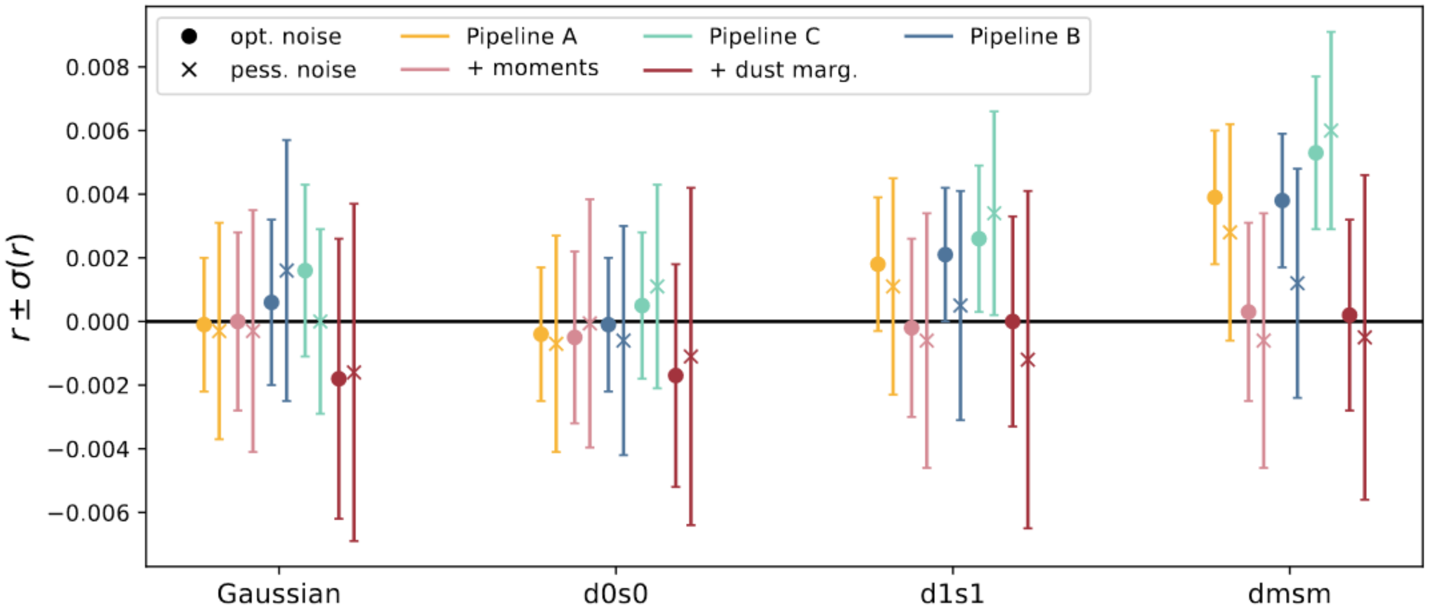
(Wolz et al., 2023)

Minimum variance

(Wolz et al., 2023, SO Collab., 2019)

Q and U observed multi-frequency maps → **E-B leakage** → **B-mode multi-frequency maps** → **NILC** → **NILC B-mode CMB map** → **B-mode power spectrum**

APPLICATION TO SO-SATs ($\delta r \sim 0.003$)

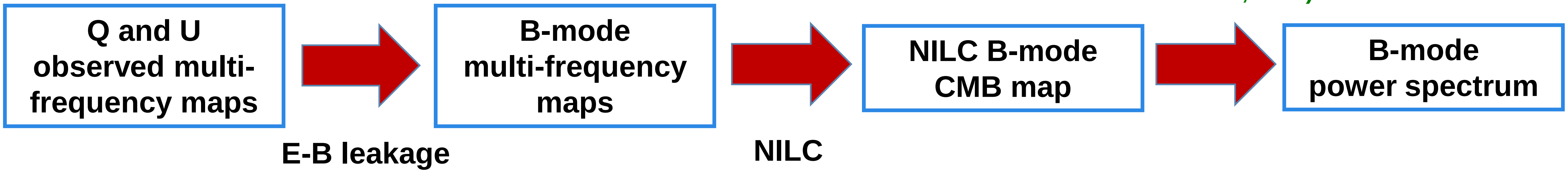


(Wolz et al., 2023)

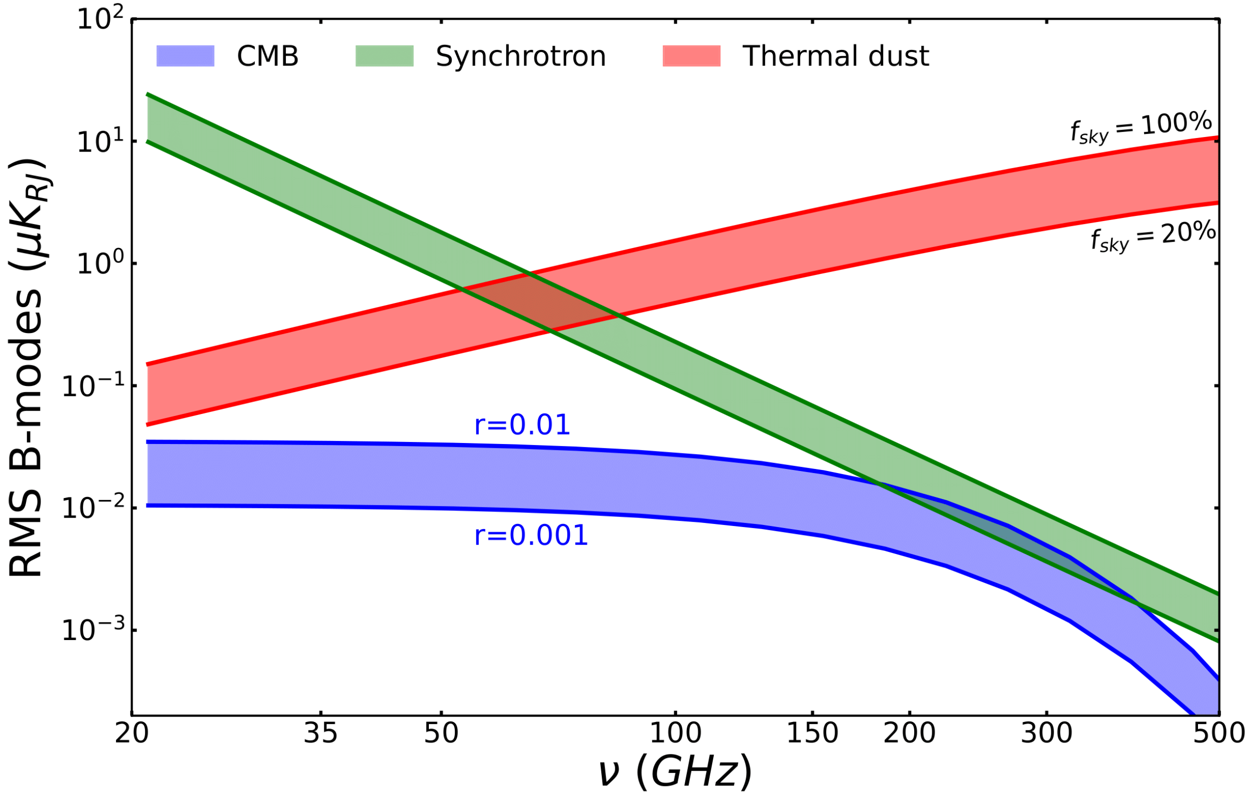
Minimum variance

(Carones et al., 2023b)

(Wolz et al., 2023, SO Collab., 2019)



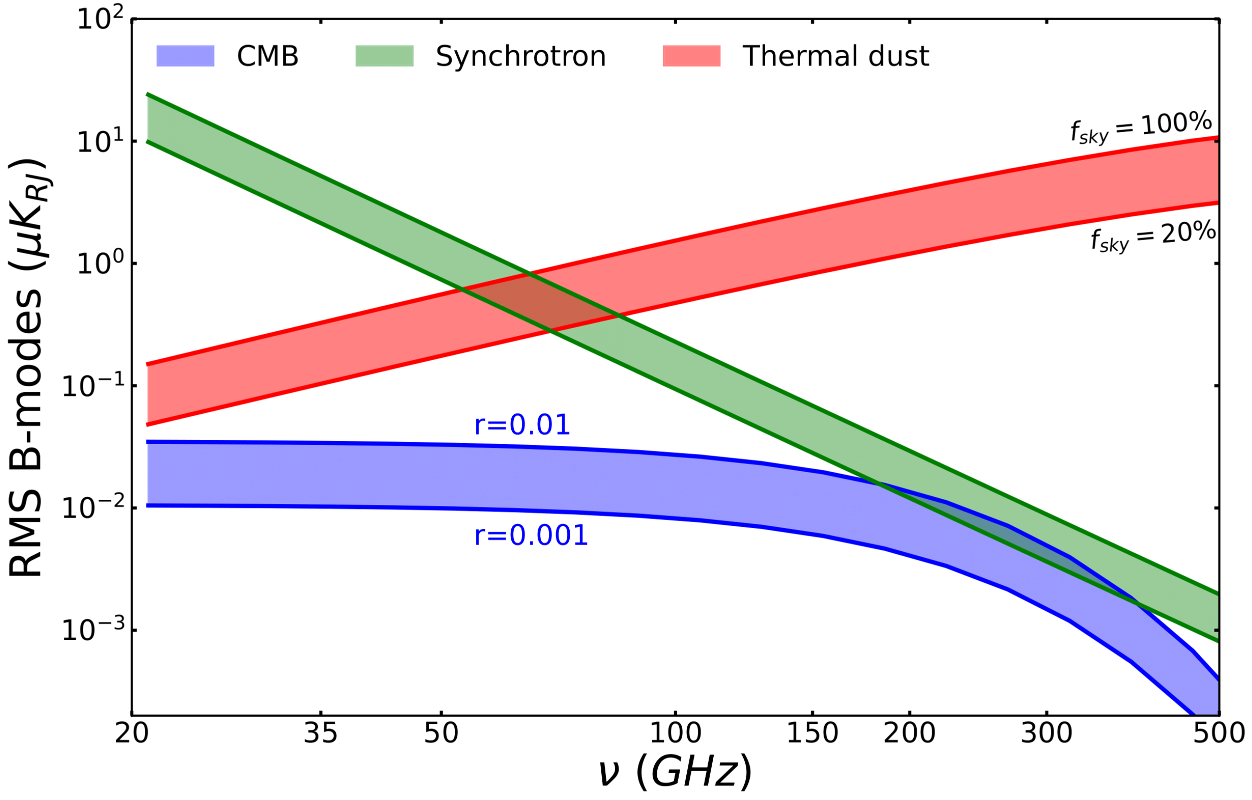
CONCLUSIONS



- All over the sky, B-mode foregrounds at their minimum are at a level $r \sim [0.05 - 1.5]$ (Krachmalnicoff et al., A&A 588, A65, 2016)
- Future CMB experiments need effective component separation methods to achieve the targeted sensitivity

- Complementarity between minimum variance and parametric is fundamental
- Component separation can benefit from optimisation of domains, inclusion of moments' fitting/deprojection,…

CONCLUSIONS



- All over the sky, B-mode foregrounds at their minimum are at a level $r \sim [0.05 - 1.5]$ (Krachmalnicoff et al., A&A 588, A65, 2016)
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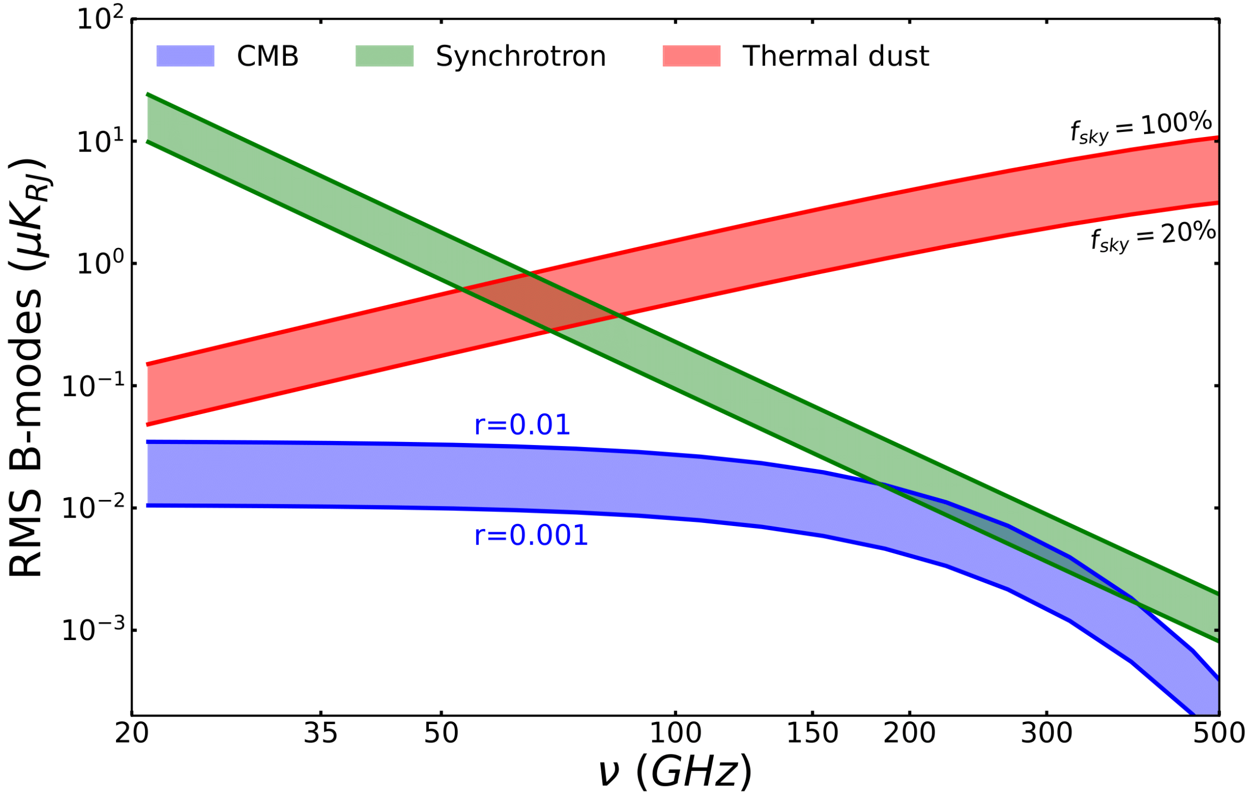

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THANK YOU FOR THE ATTENTION